108 Maximum Sum

Background

A problem that is simple to solve in one dimension is often much more difficult to solve in more than one dimension. Consider satisfying a boolean expression in conjunctive normal form in which each conjunct consists of exactly 3 disjuncts. This problem (3-SAT) is NP-complete. The problem 2-SAT is solved quite efficiently, however. In contrast, some problems belong to the same complexity class regardless of the dimensionality of the problem.

The Problem

Given a 2-dimensional array of positive and negative integers, find the sub-rectangle with the largest sum. The sum of a rectangle is the sum of all the elements in that rectangle. In this problem the sub-rectangle with the largest sum is referred to as the *maximal sub-rectangle*. A sub-rectangle is any contiguous sub-array of size 1×1 or greater located within the whole array. As an example, the maximal sub-rectangle of the array:

$$\begin{array}{rrrr} 0 & -2 & -7 & 0 \\ 9 & 2 & -6 & 2 \\ -4 & 1 & -4 & 1 \\ -1 & 8 & 0 & -2 \end{array}$$

is in the lower-left-hand corner:

$$\begin{array}{rr} 9 & 2 \\ -4 & 1 \\ -1 & 8 \end{array}$$

and has the sum of 15.

Input and Output

The input consists of an $N \times N$ array of integers. The input begins with a single positive integer N on a line by itself indicating the size of the square two dimensional array. This is followed by N^2 integers separated by white-space (newlines and spaces). These N^2 integers make up the array in row-major order (i.e., all numbers on the first row, left-to-right, then all numbers on the second row, left-to-right, etc.). N may be as large as 100. The numbers in the array will be in the range $[-127, 127]$.

The output is the sum of the maximal sub-rectangle.

Sample Input

```
4
0 -2 -7  0 9  2 -6  2
-4  1 -4  1 -1
8  0 -2
```

Sample Output

```
15
```