

Math 2283 - Introduction to Logic

Quiz #18 - 2016.02.26 Solutions

1. Prove (or disprove) that the two sentential functions: $p \rightarrow q$ and $(p \vee q) \rightarrow q$ are equivalent.

We prove using a truth table:

| p | q | $p \rightarrow q$ | $p \vee q$ | $(p \vee q) \rightarrow q$ | $[(p \vee q) \rightarrow q] \leftrightarrow (p \rightarrow q)$ |
|-----|-----|-------------------|------------|----------------------------|--|
| T | T | T | T | T | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | T | T |

2. Define $U = \mathbb{N}$ to be the set of all positive integers, and $\mathbb{K} = \{6, 12, 18, \dots\}$. Determine *all* values which make the following sentential function true.

$$\forall x \in \mathbb{K} (x/y \in \mathbb{N} \wedge y \in \mathbb{K}')$$

We are essentially looking for all positive integers which evenly divide into all positive multiples of 6, not including. The values of $y = 1$, $y = 2$, and $y = 3$ make this sentential function true (note that y is the only not bound variable in the function).