

Math 4133 - Linear Algebra

Quiz #9 - 2015.02.05

Solutions

1. Compute the determinant of the following matrix:

$$A = \begin{bmatrix} -6 & -2 & 1 \\ 5 & 3 & -2 \\ 0 & -2 & 1 \end{bmatrix}$$

We will take the determinant along the last row:

$$\begin{aligned} \det \left(\begin{bmatrix} -6 & -2 & 1 \\ 5 & 3 & -2 \\ 0 & -2 & 1 \end{bmatrix} \right) &= (-1)^{3+2} \cdot (-2) \cdot \det \left(\begin{bmatrix} -6 & 1 \\ 5 & -2 \end{bmatrix} \right) + (-1)^{3+3} \cdot (1) \cdot \det \left(\begin{bmatrix} -6 & -2 \\ 5 & 3 \end{bmatrix} \right) \\ &= (-1) \cdot (-2) \cdot (7) + (1) \cdot (1) \cdot (-8) \\ &= 6 \end{aligned}$$

2. After converting a system of equations to augmented matrix form and performing row operations on said matrix, the row-reduced matrix is given by:

$$\begin{bmatrix} 1 & 0 & 0 & 3 & -4 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Express the solution to the original system of equations using this new, but equivalent matrix.

The three equations are:

$$\begin{aligned} x_1 + 3x_4 &= -4, \\ x_2 + 2x_4 &= 1, \\ x_3 - 2x_4 &= 0 \end{aligned}$$

So our solution is

$$(x_1, x_2, x_3, x_4) = (-3x_4 - 4, -2x_4 + 1, 2x_4, x_4)$$