Math 2283 - Introduction to Logic

Quiz #21 - 2016.03.07 Solutions

For this quiz, let the universal set U by the set of natural numbers, that is, $U = \{1, 2, 3, \ldots\}$. Furthermore, define the following relations G, N, and T as follows:

$$xGy \overset{def}{\longleftrightarrow} x \text{ and } y \text{ have no common factors greater than 1}$$

$$xNy \overset{def}{\longleftrightarrow} x \text{ and } y \text{ have exactly one common factor greater than 1}$$

$$xTy \overset{def}{\longleftrightarrow} x \text{ and } y \text{ have at least one common factor greater than 1}$$

1. Which of the above relations are reflexive?

   The relation T is reflexive. (Although what about $1T1$?)

2. Which of the above relations are symmetrical?

   All three of the relations are symmetrical.

3. Which of the above relations are transitive?

   None of the relations are transitive.

4. Is $G' = T$?

   If it is not true that x and y have no common factors greater than 1, then they must have at least 1 common factor, which is the definition of T. Thus the answer is yes!

5. Is $N \subseteq T$?

   If x and y have exactly one common factor greater than 1, then x and y have at least one common factor greater than 1. Thus, $N \subseteq T$.