Math 2283 - Introduction to Logic

Quiz #22 - 2016.03.09 Solutions

For the following proof, please justify each of the steps given.

Theorem: $(K \cap L)' = K' \cup L'$

Proof:

(1)	$x \in (K \cap L)' \leftrightarrow \sim x \in K \cap L$	Instance of Definition of $'$, $K : K \cap L$
(2)	$x \in K \cap L \leftrightarrow (x \in K \wedge x \in L)$	Definition of $K \cap L$
(3)	$\sim x \in K \cap L \leftrightarrow \sim (x \in K \wedge x \in L)$	Biconditional of (2) with negation
(4)	$x \in (K \cap L)' \leftrightarrow \sim (x \in K \wedge x \in L)$	Substitute (3) into (1)
(5)	$\sim (x \in K \wedge x \in L) \leftrightarrow \sim x \in K \vee \sim x \in L$	Instance of: $\sim (p \wedge q) \leftrightarrow \sim p \vee \sim q$, $p : x \in K$, $q : x \in L$
(6)	$x \in (K \cap L)' \leftrightarrow \sim x \in K \vee \sim x \in L$	Substitute (5) into (4)
(7)	$\sim x \in K \leftrightarrow x \in K'$	Definition of $'$
(8)	$\sim x \in L \leftrightarrow x \in L'$	Instance of Definition of $'$, $K : L$
(9)	$x \in (K \cap L)' \leftrightarrow (x \in K' \vee x \in L')$	Substitute (8) and (7) into (6)
(10)	$(x \in K' \vee x \in L') \leftrightarrow x \in K' \cup L'$	Instance of Definition of $\cup$, $K : K'$, $L : L'$
(11)	$x \in (K \cap L)' \leftrightarrow x \in K' \cup L'$	Substitute (10) into (9)

□

Comment: Step 3 really is two steps in 1, if you have $p \leftrightarrow q$, then you also have $\sim p \leftrightarrow \sim q$.