Math Subject GRE Questions

Calculus and Differential Equations

1. Find the sum of the series: $\sum_{n=1}^{\infty} \frac{n}{4^{n+1}}$

(A) $\frac{1}{12}$ (B) $\frac{1}{9}$ (C) $\frac{1}{6}$ (D) $\frac{4}{3}$ (E) $\frac{1}{3}$

2. Let $f(x) = x^{2n} - x^{2n-1} + \ldots + x^4 - x^3 + x^2 - ax + b$. Which value for the pair (a, b) will insure that the x-axis will be tangent to the graph of $f(x)$ at $x = 1$?

(A) $(1, 1)$ (B) $(n, n-1)$ (C) $(n-1, n)$ (D) $(n+1, n)$ (E) $(n, n+1)$

3. Given that the Taylor series for $g(x)$ is

$$\sum_{n=0}^{\infty} a_n x^n,$$

and $f(x) = (g(x))^3$, find $f'''(0)$.

(A) $3a_0\left(2a_1^2 + a_0a_2\right)$ (B) $6a_0\left(a_1^2 + a_0a_2\right)$ (C) $3a_0\left(2a_1 + a_0a_2\right)$ (D) $6a_0\left(a_1 + a_0a_2\right)$
(E) None of these

4. Evaluate the following limit:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n\left(1 + \frac{k}{n}\right)}$$

(A) 1 (B) $\ln(2)$ (C) 0 (D) ∞ (E) Does not exist

5. Define a function, $f(x)$, to be *differentiably redundant* of order n if the n-th derivative $f^{(n)}(x) = f(x)$ but $f^{(k)}(x) \neq f(x)$ for $k < n$. For easy examples, in this context, e^x is of order 1, e^{-x} is of order 2, and $\cos(x)$ is of order 4. Which of the following functions is differentiably redundant of order 6?

(A) $e^{-x} + e^{-x/2}\cos\left(\frac{\sqrt{3}}{2}x\right)$ (B) $e^{-x} + \cos(x)$ (C) $e^{x/2}\sin\left(\frac{\sqrt{3}}{2}x\right)$
(D) Both (A) and (C) (E) (A), (B), and (C)

6. In the partial fractions decomposition of

$$\frac{s^2+1}{\left(s^2-2\right)\left(s^2+3\right)},$$

the numerator of the fraction with denominator $s^2 + 3$ is

(A) 3 (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $2s+1$ (E) $1-2s$

7. Determine the value of k is x^k a solution of the following differential equation:

$$x^2y'' - 3xy' - 4y = 0$$

(A) 4 (B) 3 (C) 2 (D) 1 (E) None of these

8. The surface given by $z = x^2 - y^2$ is cut by the plane given by $y = 3x$, producing a curve in the plane. Find the slope of this curve at the point $(1, 3, -8)$.

(A) 3 (B) -16 (C) $-8\sqrt{\frac{2}{5}}$ (D) 0 (E) $\frac{18}{\sqrt{10}}$

9. If $f(x) = \int_1^{x^2} \frac{1}{1+t^3}\,dt$, then $f'(2)$ is:

(A) $\frac{4}{65}$ (B) $\frac{1}{9}$ (C) $\ln\left(\frac{65}{2}\right)$ (D) $\ln\left(\frac{9}{2}\right)$ (E) 0.23

10. The series $\sum_{n=2}^{\infty} \frac{1}{n \cdot 3^n}$ must:

(A) converge to a value greater than 1/4
(B) converge to a value greater than 1/9
(C) converge to a value less than 1/18
(D) converge to a value less than 1/12
(E) diverge

11. The volume, V, of the region in space bounded above by the surface $x^2+y^2+z^2 = 4$ and below by $z = -\sqrt{x^2+y^2}$ is represented by a triple integral in spherical coordinates as

$$\iiint_V \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$$

Find the upper limit of integration for ϕ.

(A) π (B) $\frac{3}{4}\pi$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{6}$

12. If $f(x) = \ln(x)$, find

$$\lim_{m\to 0}\left[\lim_{n\to 0} \frac{f(2+m+n) - f(2+m) - f(2+n) + f(2)}{mn}\right]$$

(A) $\frac{1}{4}$ (B) 1 (C) -1 (D) $\frac{1}{2}$ (E) $-\frac{1}{4}$

13. Evaluate the following product:

$$\prod_{k=2}^{\infty}\left(1 - \frac{1}{k^2}\right)$$

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{1}{8}$

14. Evaluate the integral

$$\int_0^1 \left(\ln\left(\frac{1}{x}\right)\right)^5 dx$$

(A) 120 (B) ∞ (C) 1 (D) 720 (E) 24

15. Evaluate the integral

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos(x)}{\cos(x)+\sin(x)}\, dx$$

(A) 1 (B) $\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{4}$ (E) π

16. The domain of $f(x) = \int (x + 2x^2 + 3x^3 + \ldots)\, dx$ is

(A) $(-1, 1)$ (B) $[-1, 1)$ (C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right)$ (E) $(-1, 1]$

17. Find the slope of the tangent line to the ellipse

$$2x^2 + y^2 + 30 = 8y - 12x$$

at (x_0, y_0) where $x_0 = -2$ and $y_0 > 4$.

(A) $\frac{1}{\sqrt{2}}$ (B) $-\sqrt{2}$ (C) 2 (D) $\frac{1}{2}$ (E) -2

18. Given

$$p(x) = \sum_{n=1}^{\infty} \frac{(x-2)^k}{k^2},$$

find the interval in which $p'(x)$ converges.

(A) $\{2\}$ (B) $[1, 3)$ (C) $[1, 3]$ (D) $(1, 3]$ (E) $\mathbb{R}$

19. The integral

$$\int_{-24}^{4} \frac{1}{\sqrt[3]{(x-3)^2}}\, dx$$

(A) converges to 6 (B) diverges to ∞ (C) converges to 9 (D) converges to 12
(E) diverges to $-\infty$

20. Given that f is a differentiable function with normal line $4x + y = 9$ at $x = 2$ then

$$\lim_{x \to 0} \frac{f(2-x) - f(2+x)}{x}$$

is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) Undefined

Abstract Algebra/Topology

1. Find the index of the subgroup generated by the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$$

in the alternating group A_5.

(A) 20 (B) 40 (C) 24 (D) 3 (E) 5

2. Let the set S be infinite and let the set T be countably infinite. Let $\overline{S}$ denote the complement of S. If S and T are both subsets of the real numbers, which of the following pairs of sets must be of the same cardinality?

(A) T, $S \cap T$ (B) S, $S \cup T$ (C) T, $\overline{S} \cup T$ (D) Both (A) and (B)
(E) Both (A) and (C)

3. How many topologies are possible on a set of 2 points?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

4. Find the remainder on dividing 3^{20} by 7.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

5. A subgroup H in group G is invariant if $gH = Hg$ for every $g \in G$. If H and K are both invariant subgroups of G, which of the following is also an invariant subgroup?

(A) $H \cap K$ (B) HK (C) $H \cup K$ (D) Both (A) and (B) (E) Both (B) and (C)

6. Let R be a ring such that $x^2 = x$ for all $x \in R$. Which of the following must be true?

(A) $x = -x$ for all $x \in R$ (B) R is commutative (C) $xy + yx = 0$ for all $x, y \in R$
(D) Both (A) and (C) (E) (A), (B), and (C)

7. In the integral domain $D = \left\{ r + s\sqrt{17} \mid r, s \in \mathbb{Z} \right\}$, which of the following is irreducible?

(A) $8 + 2\sqrt{17}$ (B) $3 - \sqrt{17}$ (C) $9 - 2\sqrt{17}$ (D) $7 + \sqrt{17}$ (E) $13 + \sqrt{17}$

8. In the finite field $\mathbb{Z}_{17}$, the multiplicative inverse of 10 is

(A) 13 (B) 12 (C) 11 (D) 9 (E) 7

9. Let f be a mapping from a topological space X onto itself. Which of the following is true for continuous f?

(A) Every open set in X is the image of an open set in X.
(B) $f^{-1}(B)$ is bounded for each bounded set $B \subseteq X$.
(C) f is one-to-one.
(D) Both (A) and (B).
(E) Both (A) and (C).

10. Let R be a ring and let $x \neq 0$ be a fixed element of R. Which of the following is a subring of R?

(A) $\{ r \,|\, xr = 0\}$ (B) $\{ x \,|\, x^{-1} \in R\}$ (C) $\{ x^n \,|\, n = 1, 2, \ldots\}$
(D) $\{ nx \,|\, n \in \mathbb{Z}\}$ (E) Both (A) and (D)

11. The number of solutions of $p(x) = x^2 + 3x + 2$ in $\mathbb{Z}_6$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

12. Find the characteristic of the ring $\mathbb{Z}_2 + \mathbb{Z}_3$.

(A) 0 (B) 6 (C) 3 (D) 4 (E) 2

13. The factor group $\dfrac{\mathbb{Z}_2 \times \mathbb{Z}_3}{\langle(0, 1)\rangle}$ has order:

(A) 2 (B) 3 (C) 4 (D) 1 (E) 6

14. Let $X = \{a, b, c\}$. Which of the following classes of subsets of X does not form a topology on X?

(A) $\{ X, \emptyset \}$ (B) $\{ X, \emptyset, \{a\} \}$
(C) $\{ X, \emptyset, \{a\}, \{b\}, \{a, b\} \}$ (D) $\{ X, \emptyset, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$
(E) $\mathcal{P}(X)$, the power set of X

15. Determine the number of homomorphisms from the group $\mathbb{Z}_8$ onto the group $\mathbb{Z}_4$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. The remainder of 5^{34} when divided by 17 is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

17. Which of the following is *not* a proper ideal of the ring $\mathbb{Z}_{12}$?

(A) $\langle 5\rangle$ (B) $\langle 8\rangle$ (C) $\langle 2\rangle$ (D) $\langle 3\rangle$ (E) $\langle 4\rangle$

18. Find the number of units in the ring $\mathbb{Z}_5$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. Let F be a finite field, then which of the following can be the cardinality of F?

(A) 21 (B) 45 (C) 27 (D) 14 (E) 33

20. In the permutation group S_5, find the product of the elements $(21453) \cdot (35214)$.

(A) (41352) (B) (43215) (C) (43152) (D) (43125) (E) (34251)

Linear Algebra

1. Given that S and T are subspaces of a vector space, which of the following is also a subspace?

(A) $S \cap T$ (B) $S \cup T$ (C) $2S$ (D) Both (A) and (C) (E) Both (B) and (C)

2. In a homogeneous system of 5 linear equations in 7 unknowns, the rank of the coefficient matrix is 4. The maximum number of independent solution vectors is:

(A) 5 (B) 2 (C) 4 (D) 1 (E) 3

3. If A is an $n \times n$ matrix with diagonal entries, a, and other entries, b, then one eigenvalue of A is $a - b$. Find another eigenvalue of A.

(A) $b - a$ (B) $nb + a - b$ (C) $nb - a + b$ (D) 0 (E) None of these

4. Find k so that the matrix

$$\begin{bmatrix} k & 1 & 2 \\ 1 & 2 & k \\ 1 & 2 & 3 \end{bmatrix}$$

has eigenvalue $\lambda = 1$.

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) 1 (E) -1

5. If A is a matrix of square dimension $n \geq 4$, and $a_{i,j} = i + j$ represents the entry in row i and column j, then the rank of A is always:

(A) 1 (B) 2 (C) $n - 2$ (D) $n - 1$ (E) None of these

6. Which of the following matrices is normal?

(A) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & i \\ -1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} i & 1 \\ -1 & 0 \end{bmatrix}$ (E) $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T(x, y) = \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$$

Find the adjoint of T.

(A) $\begin{bmatrix} 2x + y \\ -x + 3y \end{bmatrix}$ (B) $\begin{bmatrix} x + 2y \\ x - 3y \end{bmatrix}$ (C) $\begin{bmatrix} 2x + y \\ x - 3y \end{bmatrix}$ (D) $\begin{bmatrix} \frac{x}{2} - y \\ -x + \frac{y}{3} \end{bmatrix}$ (E) $\begin{bmatrix} 3x + y \\ -x + 2y \end{bmatrix}$

8. Given $T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, the sum of the elements in T^n is:

(A) $3n$ (B) $n + 3$ (C) n (D) $2n$ (E) $n + 2$

9. Let $b : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ be a bilinear form defined by

$$b(\overrightarrow{x}, \overrightarrow{y}) = x_1y_1 - 2x_1y_2 + x_2y_1 + 3x_2y_2$$

where $\overrightarrow{x} = \langle x_1, x_2\rangle$ and $\overrightarrow{y} = \langle y_1, y_2\rangle$. Find the 2×2 matrix B relative to the basis $U = \{\overrightarrow{u_1}, \overrightarrow{u_2}\}$ where $\overrightarrow{u_1} = \langle 0, 1\rangle$ and $\overrightarrow{u_2} = \langle 1, 1\rangle$.

(A) $\begin{bmatrix} 5 & -3 \\ 0 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}$ (E) $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

10. For matrices

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 3 \\ 0 & -2 \end{bmatrix}$$

the matrix D is a linear combination $(aA + bB + cC)$ of A, B, C for a, b, c given by

(A) $1, 1, -1$ (B) $2, 1, -1$ (C) $2, 2, -2$ (D) $1, -2, 1$ (E) $-1, 1, -2$

11. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(\langle x, y, z\rangle) = \langle x + y, x - y + z, y + 2z\rangle.$$

Find the trace of T.

(A) 5 (B) -1 (C) 0 (D) 7 (E) 2

12. Given that $\langle 1, 2, 3\rangle$ is an eigenvector for the matrix

$$\begin{bmatrix} 2 & 3 & -1 \\ 3 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix},$$

find the corresponding eigenvalue.

(A) 5 (B) 4 (C) 3 (D) 2 (E) None of the above

13. Given that a 3×3 matrix A has only one eigenvalue, what is the dimension of the corresponding eigenspace?

(A) 1 (B) 2 (C) 3 (D) 1 or 2 (E) 1, 2, or 3

14. Define a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ by $\langle x, y\rangle T = \langle x + y, x - y, x\rangle$. Define S, a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$, to be an inverse of T if $(\langle x, y\rangle T)\, S = \langle x, y\rangle$. Which of the following represent an inverse of T?

(A) $\langle x, y, z\rangle S = \left\langle z, \dfrac{x}{2} - \dfrac{y}{2}\right\rangle$ (B) $\langle x, y, z\rangle S = \left\langle \dfrac{x}{2} + \dfrac{y}{2}, \dfrac{x}{2} - \dfrac{y}{2} - z\right\rangle$

(C) $\langle x, y, z\rangle S = \left\langle \dfrac{x}{3} + \dfrac{y}{3} + \dfrac{z}{3}, x - y\right\rangle$ (D) both (A) and (B)

(E) all three

15. Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 1 & 5 \end{bmatrix}$$

and $B = A^{-1}$, find the entry in row 3 and column 2 of B.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) $-\frac{1}{2}$ (E) $-\frac{1}{4}$

16. The column space of a 5×6 matrix is spanned by the vectors $\langle 1,0,0,0,0\rangle$, $\langle 0,0,1,0,0\rangle$, and $\langle 2,0,3,0,0\rangle$. Find the dimension of the solution space of the matrix.

(A) 3 (B) 3 (C) 6 (D) 2 (E) 5

17. If T is a linear transformation mapping vectors $\langle 1,0,0\rangle$, $\langle 0,1,0\rangle$, and $\langle 0,0,1\rangle$ to the vectors $\langle 1,2,3\rangle$, $\langle 2,3,1\rangle$, and $\langle 1,1,-2\rangle$ respectively, which vector is in the image of the vector $\langle 3,-2,1\rangle$ under T?

(A) $\langle 1,1,7\rangle$ (B) $\langle 1,0,5\rangle$ (C) $\langle 0,1,5\rangle$ (D) $\langle 0,1,9\rangle$ (E) $\langle 1,7,0\rangle$

18. If $\det(A) = 3$ and $\det(B) = 2$, find the determinant of the matrix $2(AB)^{-1}$ if A and B are 4×4 matrices.

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$ (E) 12

19. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and let I be an identity matrix. Which matrix polynomial is zero?

(A) $A^2 - 10A + I$ (B) $A^2 - 10A$ (C) $A^2 - 5A - 2I$ (D) $A^2 + 5A - 2I$ (E) $A^2 + 5A + 2I$

20. Which of the following is an eigenvalue of the matrix

$$A = \begin{bmatrix} 1 & 3 & 3 & 3 \\ 3 & 1 & 3 & 3 \\ 3 & 3 & 1 & 3 \\ 3 & 3 & 3 & 1 \end{bmatrix}$$

(A) -1 (B) -2 (C) 1 (D) 2 (E) 0

Complex Analysis

1. Find the point $x + iy$ at which the complex function $f(x + iy) = x^2 + y^2 + 2xi$ is differentiable.

(A) 0 (B) i (C) $1 - i$ (D) $1 + i$ (E) $-i$

2. Express z^{14} in the form $a + bi$ if $z = \dfrac{1+i}{\sqrt{2}}$.

(A) $-i$ (B) -1 (C) 1 (D) i (E) $\dfrac{1+i}{128}$

3. Evaluate the following limit:

$$\lim_{n\to\infty}\left(\sqrt{n^4 + in^2} - n^2\right)$$

(A) $\dfrac{i}{2}$ (B) 0 (C) ∞ (D) $-\dfrac{1}{2}$ (E) $\sqrt{i}$

4. Let $\mathcal{C}$ be the circle $|z| = 3$, described in counterclockwise orientation, and write

$$g(w) = \int_{\mathcal{C}} \frac{2z^2 - 2 - z}{z - w}\, dw$$

Evaluate $g(2)$.

(A) 1 (B) $2\,\pi\, i$ (C) 0 (D) $4\,\pi\, i$ (E) $8\,\pi\, i$

5. Which of the following functions is (are) analytic?

I. $\overline{z}$ II. $\overline{z}\sin(z)$ III. $z + \sin(z)$ IV. $z + \overline{z}$ V. ze^z

(A) I only (B) I and II only (C) III and V only (D) IV only (E) None of the above

6. If w is an n^{th} root of unity other than one, then the sum $w + w^2 + \ldots + w^{n-1}$ is equal to

(A) 1 (B) 0 (C) -1 (D) 3 (E) -2

7. Let $u(x, y)$ be harmonic in a domain D and $v(x, y)$ is an harmonic conjugate of u. Let $f(z) = u(x, y) + i\, v(x, y)$. Which statements are true?

I. $g(z) = v - i\, u$ is analytic in D.
II. $f'(z) = u_x + i\, v_y$.
III. $v(x, y) + x + y$ satisfies Laplace's equation in D.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

8. Determine the complex roots of the equation $e^{3z} + i = 0$.

(A) $\dfrac{i}{3}\left(-\dfrac{\pi}{2} + 2\, n\, \pi\right)$, n an integer (B) $\dfrac{i}{3}\left(\dfrac{\pi}{2} + 2\, n\, \pi\right)$, n an integer

(C) $\frac{i}{3}\left(\frac{\pi}{2}+n\,\pi\right)$, n an integer (D) $\frac{i}{2}\left(\frac{\pi}{2}+2\,n\,\pi\right)$, n an integer
(E) $\frac{i}{3}\,(\pi+2\,n\,\pi)$, n an integer

9. Determine the Laurent series for $f(z) = 1/(z-2)$ which converges in the annulus $1 < |z-3| < \infty$.

(A) $\sum_{n=0}^{\infty}(z-3)^n$ (B) $\sum_{n=0}^{\infty}(z-3)^{-n}$ (C) $\sum_{n=0}^{\infty}(-1)^n(z-3)^{-n-1}$
(D) $\sum_{n=0}^{\infty}(-1)^n(z-3)^{-n}$ (E) $\sum_{n=1}^{\infty}(-1)^n(z-3)^{-n}$

10. Let $f(z) = \frac{1}{z^2+1}$. Evaluate $\int_{\mathcal{C}} f(z)\,dz$ where $\mathcal{C}$ is a simple, closed, counterclockwise oriented curve which contains in its interior the points $\pm i$ and $\pm i$.

(A) 0 (B) π (C) $\pi\,i$ (D) $2\,\pi$ (E) $2\,\pi, i$

11. Determine an harmonic conjugate $v(x,y)$ for the harmonic function $u(x,y) = y + 3xy^2 - x^3$.

(A) $-y+3xy^2-x^3$ (B) $x+3xy^2-x^3$ (C) y^3-3x^2y-x (D) y^3-3x^2y+x (E) y^3-3x^2y+x

12. Suppose $f(z)$ is a nonconstant entire function. Which statement is always true?

(A) $\lim_{z\to\infty} f(z) = 0$ (B) $\lim_{z\to 0} f(z) = 0$ (C) $f'(z)$ may not be entire
(D) $\int_{\mathcal{C}} f(z)\,dz = 2\,\pi\,i$ for every simple closed curve in the complex plane
(E) None of the above

13. If $z = i^i$, then z can assume the real value

(A) -1 (B) 1 (C) $e^{\frac{3}{2}\pi}$ (D) $e^{2\pi}$ (E) $\frac{\pi}{2}$

14. The sum of the 9^{th} roots of unity is

(A) 0 (B) 1 (C) 9 (D) 10 (E) 1+i

15. The fixed point(s) of the Mobius transformation $w(z) = \frac{z-2}{z-1}$ is (are)

(A) $1 \pm \sqrt{3}$ (B) $1 \pm i$ (C) $2i$ (D) $1 \pm 2i$ (E) $-1 \pm \sqrt{2}i$

16. The value of

$$\int_{\mathcal{C}} \frac{\cos(z)}{z(z-\pi)}\,dz$$

where $\mathcal{C}$ is the circle $|z-1| = 2$ is

(A) 0 (B) $2i$ (C) $-2i$ (D) $-4i$ (E) $4i$

17. Evaluate the sum

$$\sum_{j=0}^{10} (-i)^j$$

(A) i (B) -1 (C) $-i$ (D) $1+i$ (E) $1-i$

18. The function $f(z) = \sin(x)\cosh(y) + v(x, y)$ is analytic for $v(x, y)$ equal to

(A) $\cos(x)\cosh(y)$ (B) $\cos(x)\sinh(y)$ (C) $-\sin(y)\cosh(x)$ (D) $\sin(x)\sinh(y)$
(E) $\sin(y)\cosh(x)$

19. Find the square roots of $5 - 12i$.

(A) $2 \pm 3i$ (B) $\pm 3 - 2i$ (C) $2 \pm 2i$ (D) $3 \pm 3i$ (E) None of the above

20. The equation of the circle in the complex plane passing through the points $1-i$, $2i$, and $1+i$ is

(A) $|z| = \sqrt{5}$ (B) $|z| = 5$ (C) $|z-1| = \sqrt{5}$ (D) $|z+1| = 5$ (E) $|z+1| = \sqrt{5}$

Probability and Statistics

1. Find the number of distinguishable permutations of six colored blocks if one is red, two are yellow, and three are blue.

(A) 360 (B) 60 (C) 720 (D) 120 (E) 240

2. A biased coin is tossed repeatedly until the first "tail" occurs. The expected number of tosses required to produce the first tail is estimated as T. Assuming this is true, find the probability of at least two tails in $3T$ tosses.

(A) $\dfrac{T^{3T}-(T-1)^{3T-1}(4T)}{T^{3T}}$ (B) $\dfrac{T^{3T}-(T-1)^{3T-1}(3T)}{T^{3T}}$
(C) $\dfrac{T^{3T}-(T-1)^{3T-1}(3T-1)}{T^{3T}}$ (D) $\dfrac{T^{3T}-(T-1)^{3T-1}(4T-1)}{T^{3T}}$
(E) None of these.

3. Assuming that a person selects an answer to each of the first ten questions on this examination at random and that the selections are independent, what is the probability that he/she will guess exactly five answers correct?

(A) $\dfrac{(63)\cdot 4^6}{5^{10}}$ (B) $\dfrac{(65)\cdot 4^6}{5^{10}}$ (C) $\dfrac{4^9}{5^{10}}$ (D) $\dfrac{(61)\cdot 4^6}{5^{10}}$ (E) $\dfrac{(67)\cdot 4^6}{5^{10}}$

4. On average, a baseball player gets a hit in one out of three attempts. Assuming that the attempts are independent, what is the probability that he gets *exactly* three hits in six attempts?

(A) $\dfrac{160}{3^6}$ (B) $\dfrac{160}{3^5}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{80}{3^6}$ (E) $\dfrac{40}{3^6}$

5. The random variable, X, is discrete and uniformly distributed with values 1, 2, 3, 4, 5. The variance X is

(A) 1 (B) 2 (C) 3 (D) 4 (E) None of these

6. A regular deck of 52 cards contains 4 suits of 13 denominations: $2, 3, \ldots, 10$, J, Q, K, A. In how many ways may we select a subset of 5 cards containing exactly 3 of the same denomination? In the answer below, $\cdot$ represents multiplication as usual.

(A) $24\cdot 47\cdot 52$ (B) $47\cdot 48\cdot 52$ (C) $24\cdot 39\cdot 47$ (D) $39\cdot 47\cdot 48$ (E) $6\cdot 47\cdot 48\cdot 52$

7. The number of different ways, not counting rotations, to seat six different people around a circular table is

(A) 720 (B) 60 (C) 360 (D) 180 (E) 120

8. In a sequence of consecutive throws of a die, find the probability that six will show before a one or two.

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{1}{3}$

9. How many ways can 8 teachers be divided among 4 schools if each school must receive two teachers?

(A) 520 (B) 250 (C) 2520 (D) 225 (E) 2^4

10. For what value of c is the function

$$f(x) = \begin{cases} cxe^{-x^2} & , 0 < x < \infty \\ 0 & , \text{elsewhere} \end{cases}$$

the probability density function of a random variable X?

(A) 1 (B) 2 (C) 3 (D) π (E) 2π

11. A witness to a robbery told the police that the license number of the care contained three digits, the first of which was a 9, followed by three letters, the last of which was an A. The witness cannot remember the second and third digit, nor the first and second letter, but is positive that all the numbers were different and that the first two letters were the same, but different from the last letter. How many possible license plates match the description given by the witness?

(A) 1872 (B) 2600 (C) 2106 (D) 729 (E) 1800

12. A coin is biased so that a tail is twice as likely to occur as a head. What is the expected number of tails if the coin is tossed twice?

(A) $\frac{4}{3}$ (B) $\frac{8}{9}$ (C) $\frac{4}{9}$ (D) $\frac{1}{2}$ (E) 2

13. Let x be a random variable possessing the probability density function

$$f(x) = \begin{cases} cx & , x \in [0, 10] \\ 0 & , \text{otherwise} \end{cases}$$

where $c \in \mathbb{R}$. The probability that $x \in [1, 2]$ is

(A) $\frac{1}{100}$ (B) $\frac{3}{100}$ (C) $\frac{5}{100}$ (D) $\frac{7}{100}$ (E) $\frac{9}{100}$

14. From a group of 15 mathematics graduate school applicants, 10 are selected at random. Let P be the probability that 4 of the 5 applicants who would make the best graduate students are included in the 10 selected. Which of the following statements is true?

(A) $0 \leq P \leq \frac{1}{5}$ (B) $\frac{1}{5} < P \leq \frac{2}{5}$ (C) $\frac{2}{5} < P \leq \frac{3}{5}$ (D) $\frac{3}{5} < P \leq \frac{4}{5}$ (E) $\frac{4}{5} < P \leq 1$

15. Three balls are drawn at random from a bag containing eight red and seven yellow balls. Compute the probability of drawing either three red or three yellow balls.

(A) $\frac{8}{65}$ (B) $\frac{1}{13}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$ (E) 1

16. How many possible arrangements are there of the letters $\{a, b, c, d, e\}$ in which a and b are next to each other?

(A) 24 (B) 16 (C) 32 (D) 48 (E) 6

17. There are 100 senators in the United States Senate. How many two-member committees are possible?

(A) 9900 (B) 970200 (C) 1650 (D) 4950 (E) 3300

18. Suppose that five faulty transistors are accidentally packaged with ten reliable transistors. What is the probability that the three transistors chosen at random from this package will all be reliable?

(A) $\frac{67}{91}$ (B) $\frac{14}{91}$ (C) $\frac{24}{91}$ (D) $\frac{12}{91}$ (E) $\frac{87}{91}$

19. Suppose we mark seven numbers from 1 to 49 in a game of chance and a gambling machine chooses six winning numbers. What is the probability that we will have exactly four winning numbers.

(A) $\dfrac{\binom{7}{4}\binom{42}{2}}{\binom{49}{6}}$ (B) $\dfrac{\binom{7}{4}\binom{49}{2}}{\binom{49}{6}}$ (C) $\dfrac{\binom{7}{4}\binom{49}{6}}{\binom{42}{2}}$ (D) $\binom{7}{4}\binom{49}{6}$

(E) None of the above

20. You roll a fair six-sided die until you end up with a "6". What is the probability that it will take at most 20 rolls to get your first "6"?

(A) 0.1615 (B) 0.8385 (C) 0.026 (D) 0.974 (E) 0.5