# Math 2283 - Introduction to Logic
## Quiz #24 - 2016.03.21 Solutions

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Let $U = \{0, 1, 2, 3\}$, and consider the relation $R$ which holds true only for the following pairs $(x, y)$, where $x$ is the element from the domain, and $y$ from the counter-domain.

$$\{(0,0),(1,1),(2,2),(3,3),(2,1),(1,2)\}$$

Determine whether the relation $R$ has any of the properties: reflexive, irreflexive, symmetrical, asymmetrical, transitive, intransitive, and connected.

reflexive: the relation $R$ is reflexive, since $0R0$, $1R1$, $2R2$, and $3R3$, which takes care of all $x \in U$.

irreflexive: since $R$ is reflexive, it cannot be irreflexive.

symmetrical: we know $xRx$, for $x \in U$, $xRx \to xRx$. We also have $2R1 \to 1R2$, and $1R2 \to 2R1$. Since we have no other $xRy$, with $x \neq y$, the hypothesis is only satisfied with $x = 1$ or $x = 2$. Thus, the relation is symmetrical.

asymmetrical: since $R$ is symmetrical, it cannot be asymmetrical.

transitive: so remember the definition of transitive is $xRy \wedge yRz \to xRz$. This holds for all $x = y = z$, we also need to look at any other combinations of $x$, $y$, $z$'s which make the hypothesis true:

$$1R1 \wedge 1R2 \to 1R2,$$
$$1R2 \wedge 2R1 \to 1R1,$$
$$1R2 \wedge 2R2 \to 1R2,$$
$$2R2 \wedge 2R1 \to 2R1,$$
$$2R1 \wedge 1R1 \to 2R1,$$
$$2R1 \wedge 1R2 \to 2R2.$$

These all hold, thus the relation is indeed transitive.

intransitive: since the relation is transitive, it cannot be intransitive.

connected: the relation $R$ is not connected, since, for example, $\sim 2R3$ and $\sim 3R2$.