Math 2215 - Calculus 1

Exam #2 - 2016.09.16

Solutions

---

1. Compute the following derivative:

$$\frac{d}{dx}\frac{2}{\sin(3x)}$$

$$\begin{aligned}\frac{d}{dx}\frac{2}{\sin(3x)} &= \frac{d}{dx}2\csc(3x)\\ &= -2\csc(3x)\cot(3x)\frac{d}{dx}3x\\ &= -6\csc(3x)\cot(3x)\end{aligned}$$

2. Compute the following derivative:

$$\frac{d}{dx}\cos^3(x^2)$$

$$\begin{aligned}\frac{d}{dx}\cos^3(x^2) &= 3\cos^2(x^2)\frac{d}{dx}\cos(x^2)\\ &= 3\cos^2(x^2)\cdot\left(-\sin(x^2)\frac{d}{dx}x^2\right)\\ &= 3\cos^2(x^2)\cdot\big(-\sin(x^2)\cdot 2x\big)\\ &= -6x\cos^2(x^2)\cdot\sin(x^2)\end{aligned}$$

3. Compute the following derivative:

$$\frac{d}{d\theta}\big(\sin\big(\tan(3\theta^2+1)\big)-\theta\big)$$

$$\begin{aligned}\frac{d}{d\theta}\sin\big(\tan(3\theta^2+1)\big)-\theta &= \cos\big(\tan(3\theta^2+1)\big)\cdot\frac{d}{d\theta}\big(\tan(3\theta^2+1)\big)-1\\ &= \cos\big(\tan(3\theta^2+1)\big)\cdot\sec^2(3\theta^2+1)\cdot\frac{d}{d\theta}\big(3\theta^2+1\big)-1\\ &= \cos\big(\tan(3\theta^2+1)\big)\cdot\sec^2(3\theta^2+1)\cdot 6\theta-1\end{aligned}$$

4. Compute the following derivative:

$$\frac{d}{dx}\sqrt[4]{x+\sqrt[5]{x+\sqrt[3]{x+1}}}$$

$$\begin{aligned}\frac{d}{dx}\sqrt[4]{x+\sqrt[5]{x+\sqrt[3]{x+1}}} &= \frac{1}{4\sqrt[4]{\left(x+\sqrt[5]{x+\sqrt[3]{x+1}}\right)^3}}\cdot\frac{d}{dx}\left(x+\sqrt[5]{x+\sqrt[3]{x+1}}\right)\\ &= \frac{1}{4\sqrt[4]{\left(x+\sqrt[5]{x+\sqrt[3]{x+1}}\right)^3}}\cdot\left(1+\frac{1}{5\sqrt[5]{\left(x+\sqrt[3]{x+1}\right)^4}}\cdot\frac{d}{dx}\left(x+\sqrt[3]{x+1}\right)\right)\\ &= \frac{1}{4\sqrt[4]{\left(x+\sqrt[5]{x+\sqrt[3]{x+1}}\right)^3}}\cdot\left(1+\frac{1}{5\sqrt[5]{\left(x+\sqrt[3]{x+1}\right)^4}}\cdot\left(1+\frac{1}{3\sqrt[3]{(x+1)^2}}\cdot 1\right)\right)\end{aligned}$$

5. Consider the implicitly defined function given by the equation:

$$y^2x + y^2 = x^2 - x$$

Find *all* the points (x, y) on the curve at which there is a *vertical* tangent line.

First we apply d/dx to both sides of the equation:

$$\begin{aligned} \frac{d}{dx}\left(y^2x + y^2\right) &= \frac{d}{dx}\left(x^2 - x\right) \\ 2yy'x + y^2 + 2yy' &= 2x - 1 \\ y'(2xy + 2y) &= 2x - 1 - y^2 \\ y' &= \frac{2x - 1 - y^2}{2xy + 2y} \end{aligned}$$

Vertical tangent lines will occur where the denominator is zero in the definition of y' above, so we have

$$2xy + 2y = 0 \to 2y(x + 1) = 0.$$

By the product of zeros rule, either $2y = 0$ or $x + 1 = 0$. First setting $y = 0$ in our original equation gives

$$x^2 - x = 0,$$

so two points are $(0, 0)$ and $(1, 0)$. Setting $x = -1$ in our original equation results in $0 = 0$, which is not very informative. So we could argue that any y satisfies the equation at $x = -1$ and thus there is always a vertical tangent line at $(-1, y)$ for any y.

6. State the Mean Value Theorem.

If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

7. Compute the following derivative:

$$\frac{d}{dw}\frac{\sin(2w)\cos(w) + \sec(3w)}{1 + \tan(4w)}$$

$$\begin{aligned} &\frac{d}{dw}\frac{\sin(2w)\cos(w) + \sec(3w)}{1 + \tan(4w)} \\ &= \frac{\frac{d}{dw}\left(\sin(2w)\cos(w) + \sec(3w)\right)(1 + \tan(4w)) - \left(\sin(2w)\cos(w) + \sec(3w)\right)\frac{d}{dw}(1 + \tan(4w))}{(1 + \tan(4w)^2} \\ &= \frac{\left(2\cos(2w)\cos(w) - \sin(2w)\sin(w) + 3\sec(3w)\tan(3w)\right)(1 + \tan(4w)) - \left(\sin(2w)\cos(w) + \sec(3w)\right)\left(4\sec^2(4w)\right)}{(1 + \tan(4w))^2} \end{aligned}$$

8. The following is a graph of $f'(x)$, sketch $f(x)$ on the same graph.

   The function $f'(x)$ is in blue, while $f(x)$ is in yellow.