

Math 2215 - Calculus 1

Quiz #10 - 2016.09.26

Solutions

Approximate $\sqrt[3]{\frac{1}{3}}$ using linearization. Hint: consider $\sqrt[3]{\frac{8}{27}}$

If we let $f(x) = \sqrt[3]{x}$, then setting $x_0 = \frac{8}{27}$ we know that $f(x_0) = \frac{2}{3}$. Note that $\frac{1}{3}$ is only $\frac{1}{27}$ greater than $\frac{8}{27}$, so we will linearize $f(x)$ at x_0 .

$$f(x) = \sqrt[3]{x} \longrightarrow f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

The equation of the tangent line is

$$y = f'(x_0)(x - x_0) + f(x_0).$$

Here, $x_0 = \frac{8}{27}$, $f(x_0) = \frac{2}{3}$, and $f'(x_0) = \frac{3}{4}$. So the tangent line equation is

$$y = \frac{3}{4} \left(x - \frac{8}{27} \right) + \frac{2}{3}$$

To approximate $\sqrt[3]{\frac{1}{3}}$, we plug $x = \frac{1}{3}$ into the tangent line equation:

$$\begin{aligned} \sqrt[3]{\frac{1}{3}} &\approx \frac{3}{4} \left(\frac{1}{3} - \frac{8}{27} \right) + \frac{2}{3} \\ &\approx \frac{3}{4} \cdot \frac{1}{27} + \frac{2}{3} \\ &\approx \frac{1}{36} + \frac{2}{3} \\ &\approx \frac{25}{36} \end{aligned}$$