

Math 2215 - Calculus 1

Quiz #12 - 2016.10.03

Solutions

Compute the all critical points, intervals of increase of decrease, intervals of concavity, and inflection points for the function $f(x) = x^3 - 3x^2$.

We start by computing the first derivative:

$$f'(x) = 3x^2 - 6x = 3x(x - 2).$$

So there are critical points at $x = 0$ and $x = 2$. Since this is a parabola opening upwards, the derivative will be negative between the two roots, and positive elsewhere. Therefore, we have that $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$, and decreasing on $(0, 2)$.

Next we compute the second derivative:

$$f''(x) = 6x - 6 = 6(x - 1).$$

There is a possible inflection point at $x = 1$, and since the graph of $f''(x)$ is a line with positive slope, we have that $f''(x) > 0$ for $x > 1$ and $f''(x) < 0$ for $x < 1$. So $f(x)$ is concave up on $(1, \infty)$ and concave down on $(-\infty, 1)$ resulting in a change in concavity at $x = 1$ which is thus an inflection point.