Math 2215 - Calculus 1

Final Exam - 2016.12.05 - 11:00-13:00

Name: ______________________________

Table of Derivatives

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2} \qquad \frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\sinh^{-1}(x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}\cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}\coth^{-1}(x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}\text{sech}^{-1}(x) = -\frac{1}{x\sqrt{1-x^2}} \qquad \frac{d}{dx}\text{csch}^{-1}(x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

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1. Compute the following limit: $\lim_{x\to 0} \frac{\tan(x^2)}{4x^2}$

2. Compute the following limit: $\lim_{x\to -\infty} \frac{3e^{-x} - 4e^{2x}}{2e^{-x} + 5e^{3x}}$

3. Find an equation of the tangent line to $y = 3\tan(2x) + 1$ at $x = \frac{\pi}{8}$.

4. Compute the following derivative: $\frac{d}{dz}\sin^2\left(\tanh^{-1}(2z)\right)$

5. Compute the following derivative: $\frac{d}{dx}\frac{f(x)\cdot g(x)\cdot h(x)}{r(x)}$

6. Find an equation of the tangent line to $y^2 + x\cos(y) = 4 - x$ at $(x, y) = (2, 0)$.

7. Sketch a graph of $f(x) = \frac{x}{\sqrt{x^2+2}}$. In order to accomplish this, compute and state (a) the domain, (b) x-, y-intercepts, (c) vertical and horizontal asymptotes, (d) critical points, (e) intervals of increase and decrease, (f) inflection points, and (g) intervals of concavity.

8. Find the point on the curve $y = \cos(x)$ closest to the origin.

9. A rectangular tank with base 4 m $\times$ 2 m and height 3 m is being filled at a rate of 2 m^3/min. Determine the rate at which the height of the water in the tank is rising when the tank is half full.

10. Set up the integral(s), *but do not evaluate*, to compute the finite area bounded by the functions $y = 2x(3-x)$, $y = -2x$, and $y = 4x - 12$.

11. Set up the integral(s), *but do not evaluate*, to compute the volume of surface of revolution of the region bounded by $y = \sin(x)$ and $y = \cos(x)$ for $\frac{\pi}{4} \le x \le \frac{5\pi}{4}$ and revolved around the line

$x = -1$. You may use choose whichever method you wish for this problem.

12. Set up the integral(s), *but do not evaluate*, to compute the volume of surface of revolution of the region bounded by $y = \sin(x)$ and $y = \cos(x)$ for $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$ and revolved around the line $y = 2$. You may use choose whichever method you wish for this problem.

13. Let $f(x) = \tanh^{-1}(\sin(x))$, $g(x) = \sinh^{-1}(\tan(x))$, and $h(x) = \cosh^{-1}(\sec(x))$. Compute the derivatives $f'(x)$, $g'(x)$, and $h'(x)$ and then *simplify as fully as possible.* Finally, make a clever observation.

14. Compute the following derivative: $\frac{d}{dx} f(g(x))^{a(b(x))}$

15. Compute the following definite integral: $\int_e^{e^2} \frac{1}{t\ln(t)}\, \mathrm{d}t$

16. Compute the following integral: $\int \frac{x+1}{\sqrt{1-x^2}}\, \mathrm{d}x$

17. Compute the following integral: $\int \frac{e^z}{1+e^{2z}}\, \mathrm{d}z$

18. Compute the following integral: $\int \frac{1}{\sqrt{25+4v^2}}\, \mathrm{d}v$

19. Compute the following derivative: $\frac{d}{dr} \int_{r^2}^{\sin(r)} e^{2t} + t^2\, \mathrm{d}t$

20. State the Mean Value Theorem.