

# Math 2283 - Introduction to Logic

Exam #1 - 2017.02.01

Name: \_\_\_\_\_

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1. Determine whether each of the following are constants, sentential functions, designatory functions, or sentences.

(a)  $x$  is the largest city in Oklahoma.

(b) The largest city in Oklahoma.

(c) City  $x$  has a larger population than city  $y$ .

(d) The state capitol of  $x$ .

(e)  $\forall x$ , if  $x$  is a state, then  $x$  has a capitol.

2. Determine which of the following quantified sentences are true, and which are false. Here, you may assume that  $x$  and  $y$  belong to the set of people in this classroom.

(a)  $\exists x, y$   $x$  is at least as old as  $y$ .

(b)  $\exists x \forall y$   $x$  is at least as old as  $y$ .

(c)  $\forall y \exists x$   $x$  is at least as old as  $y$ .

(d)  $\forall x, y$   $x$  is at least as old as  $y$ .

3. For what values is the following sentential function false?

$\exists y$   $y$  is a business day after day  $x$  of a standard week.

4. For the following conditional sentences, state its (a) inverse, (b) converse, and (c) contrapositive.

If there is a state revenue failure, then funding for state agencies will be reduced.

5. State the Rule of Detachment.

6. State the Rule of Substitution.

7. State the Law of Contraposition.

8. Construct a truth table for the sentence  $(p \vee q \vee r) \leftrightarrow [\sim p \rightarrow (q \vee r)]$ .

9. Convert the following to logical symbolism:

If it is not that  $p$  or it is not the case that  $q$ , then it is not the case that:  $p$  and  $q$ .

10. Justify each of steps (4) – (7) in the following proof.

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|-----|---|-------------|
| (1) | $\sim q$  | Assumption  |
| (2) | $p \vee q$  | Assumption  |
| (3) | $(p \vee q) \rightarrow (\sim p \rightarrow q)$             | Logical Law |
| (4) | $\sim p \rightarrow q$                                      |             |
| (5) | $(\sim p \rightarrow q) \rightarrow (\sim q \rightarrow p)$ |             |
| (6) | $\sim q \rightarrow p$                                      |             |
| (7) | $p$   |             |