

Math 2315 - Calculus 2

Quiz #7 - 2017.02.06

Solutions

Compute the following integral: $\int x \ln(1+x^2) dx$

We do integration by parts with $f = x^2$ and $g = \ln(1+x^2)$, thus $f' = 2x$ and $g'(x) = \frac{2x}{1+x^2}$:

$$\int x \ln(1+x^2) dx = \frac{1}{2}x^2 \ln(1+x^2) - \int \frac{x^3}{1+x^2} dx$$

In order to perform long division on the integral on the right hand side, we need to perform long division first:

$$\begin{aligned} \frac{x^3}{1+x^2} &= x + \frac{-x}{1+x^2} \\ \int x \ln(1+x^2) dx &= \frac{1}{2}x^2 \ln(1+x^2) - \int \frac{x^3}{1+x^2} dx \\ &= \frac{1}{2}x^2 \ln(1+x^2) - \int x - \frac{x}{1+x^2} dx \\ &= \frac{1}{2}x^2 \ln(1+x^2) - \int x dx + \int \frac{x}{1+x^2} dx \\ &= \frac{1}{2}x^2 \ln(1+x^2) - \frac{1}{2}x^2 + \frac{1}{2} \ln(1+x^2) + C \end{aligned}$$