

Math 2315 - Calculus 2

Exam #2 - 2017.02.24

Name: _____

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1. Solve the following initial value problem explicitly: $y' = e^{x+y}$, $y(0) = 3$
 2. Write the parametric equations $(x, y) = (x(t), y(t))$ which represents a circle of radius 3 centered at $(-1, 1)$ such that all of the following holds simultaneously:
 - (a) $(x(0), y(0)) = (-1, 4)$,
 - (b) the curve is oriented counter-clockwise,
 - (c) it takes a total of π time units to complete a full revolution back to the point $(-1, 4)$.

3. Consider the parametric equations $(x(t), y(t)) = (\sqrt{t^2 + 3}, \sin(\pi t))$.

(a) Verify that $(x(-1), y(-1)) = (x(1), y(1))$.

(b) Find the equations of the two tangents lines to $(x(t), y(t))$ for $t = -1$ and $t = 1$.

(c) Find all values of t on the interval $[-1, 1]$ for which $(x(t), y(t))$ has a horizontal or vertical tangent line.

4. Consider the parametric equations $(x(t), y(t)) = (t \cos(t), t \sin(t))$. Verify that the arclength of the curve for $-1 \leq t \leq 1$ is given by the integral

$$\int_{-\pi/4}^{\pi/4} \sec^3(\theta) d\theta$$

You do not have to calculate this integral, just simplify your arclength formula until you end up with this integral.

5. Plot the polar curve $r = \theta \sin(2\theta)$ for $0 \leq \theta \leq 2\pi$.

6. Compute the area of the loop in the first quadrant of the graph of the polar equation $r = \theta \sin(2\theta)$ from the previous problem. You may need to make use of the trigonometric identity:

$$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$$