

# Math 2315 - Calculus 2

Quiz #16 - 2017.03.28

Solutions

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Determine whether the following series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^k \cdot \ln(k)}{3^k \cdot k}$$

Since this is an alternating series, we simply need to compute

$$\begin{aligned} \lim_{k \rightarrow \infty} \frac{2^k \cdot \ln(k)}{3^k \cdot k} &= \lim_{k \rightarrow \infty} \left[ \frac{2^k \ln(k)}{3^k k} \right] \\ &= \lim_{k \rightarrow \infty} \left[ \left( \frac{2}{3} \right)^k \frac{\ln(k)}{k} \right] \\ &= \lim_{k \rightarrow \infty} \left[ \left( \frac{2}{3} \right)^k \right] \cdot \lim_{k \rightarrow \infty} \left[ \frac{\ln(k)}{k} \right] \\ &= 0 \cdot 0 \end{aligned}$$

Since the limit is zero, the series converges. To determine absolute convergence, an application of the ratio test yields a value of  $2/3$  which is less than 1, thus the series converges absolutely.