Math 2283 - Introduction to Logic

Exam #2 - 2017.04.05

Name: ______________________________

Consider the following 9 axioms, 10 theorems (which you have already proven), and the definition of set equality '=', which is given after Theorem X.

Axiom I. $K \subseteq K$
Axiom II. $(K \subseteq L \wedge L \subseteq M) \to K \subseteq M$
Axiom III. $(K \cup L \subseteq M) \leftrightarrow (K \subseteq M \wedge L \subseteq M)$
Axiom IV. $(M \subseteq K \cap L) \leftrightarrow (M \subseteq K \wedge M \subseteq L)$
Axiom V. $K \cap (L \cup M) \subseteq (K \cap L) \cup (K \cap M)$
Axiom VI. $K \subseteq U$
Axiom VII. $\emptyset \subseteq K$
Axiom VIII. $U \subseteq K \cup K'$
Axiom IX. $K \cap K' \subseteq \emptyset$

Theorem I. $K \cup K \subseteq K$
Theorem II. $K \subseteq K \cap K$
Theorem III. $K \subseteq K \cup L \wedge L \subseteq K \cup L$
Theorem IV. $K \cap L \subseteq K \wedge K \cap L \subseteq L$
Theorem V. $K \cup L \subseteq L \cup K$
Theorem VI. $K \cap L \subseteq L \cap K$
Theorem VII. $L \subseteq M \to K \cup L \subseteq K \cup M$
Theorem VIII. $L \subseteq M \to K \cap L \subseteq K \cap M$
Theorem IX. $K \cap L \subseteq K \cap (L \cup M) \wedge K \cap M \subseteq K \cap (L \cup M)$
Theorem X. $(K \cap L) \cup (K \cap M) \subseteq K \cap (L \cup M)$

Definition I. $K = L \overset{def}{\longleftrightarrow} K \subseteq L \wedge L \subseteq K$

From these axioms, theorems, and the above definition, derive the theorems on the following page with the hints provided. Remember, you can use any of the theorems previous to prove a given theorem, just not any theorems listed after the one you are attempting to prove. You are to prove as many of these theorems, *with full justification for each step of your proof*, in the allotted time as possible. Note: you do not have to go in order.

Theorem XI. $K = K$
Hint: Let $L : K$ in Definition I and apply Axiom I.

Theorem XII. $K = L \to L = K$
Hint: In Definition I swap K and L, compare the sentence thus obtained with Definition I in its original formulation.

Theorem XIII. $(K = L \wedge L = M) \to K = M$
Hint: This theorem can be derived from Definition I and Axiom II.

Theorem XIV. $K \cup K = K$
Hint: In Definition I perform the substitution $K : K \cup K$ and $L : K$; apply Theorem I and Theorem III (with $L : K$).

Theorem XV. $K \cap K = K$
Hint: The proof is analogous to that of the preceding theorem.

Theorem XVI. $K \cup L = L \cup K$
Hint: By Theorem V we have:

$$K \cup L \subseteq L \cup K, \quad L \cup K \subseteq K \cup L$$

To these formulas apply Definition I.

Theorem XVII. $K \cap L = L \cap K$
Hint: The proof is similar to that of Theorem XVI.

Theorem XVIII. $K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$
Hint: This theorem follows from: Definition I, Axiom V, and Theorem X.

Theorem XIX. $K \cup K' = U$
Hint: This theorem can be derived with the help of Definition I, from Axiom VI (with $K : K \cup K$), and Axiom VIII.

Theorem XX. $K \cap K' = \emptyset$
Hint: Apply Definition I, Axiom VII, and Axiom IX.