Math 2315 - Calculus 2

Exam #3 - 2017.04.20

Name: ______________________________

For problems 1 through 5, determine whether each series converges absolutely, conditionally, or is divergent.

1. $\displaystyle\sum_{k=0}^{\infty}(-1)^k\left(\frac{3+2k}{5-3k}\right)^k$

2. $\displaystyle\sum_{k=1}^{\infty}2k^{-8/7}$

3. $\displaystyle\sum_{k=1}^{\infty}\frac{k+\cos(k)}{\sqrt{k^3+k}}$

4. $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{3+2k}{5-3k}$

5. $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{k}{k^2+1}$

6. Compute, exactly, the following series: $\displaystyle\sum_{k=1}^{\infty}\frac{4}{3k(k+5)}$

7. Determine the interval of convergence for the series $\displaystyle\sum_{k=0}^{\infty}\frac{k}{4^k}(x+1)^k$

8. Find a power series representation, and interval of convergence, of the function $f(x)=\dfrac{4}{6-3x}$.

9. Show that the Maclaurin series for $f(x)=(x+1)\cos(x)$ can be written as follows:

$$f(x)=(x+1)\cos(x)=1+\sum_{k=1}^{\infty}(-1)^k\left(\frac{1}{(2k)!}-\frac{1}{2(k-1)!}\right)x^{2k}$$

Here you may assume that all series converge for finite x, and may thus be broken up and rearranged as needed.

10. Using the Maclaurin series for $\tan(x)$, compute the following limit, and then verify your work by applying l'Hôpital's rule.

$$\lim_{x\to 0}\frac{\tan(2x)}{3x}$$