Math 2215 - Calculus 1

Quiz #6 - 2017.09.11

Solutions

If $f(x) = \sqrt{2x+1}$, compute $f'(x)$ by using the limit definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} \\
&= \lim_{h \to 0} \frac{(2(x+h)+1) - (2x+1)}{h \cdot (\sqrt{2(x+h)+1} + \sqrt{2x+1})} \\
&= \lim_{h \to 0} \frac{2x + 2h + 1 - 2x - 1}{h \cdot (\sqrt{2(x+h)+1} + \sqrt{2x+1})} \\
&= \lim_{h \to 0} \frac{2h}{h \cdot (\sqrt{2(x+h)+1} + \sqrt{2x+1})} \\
&= \lim_{h \to 0} \frac{h}{h} \cdot \frac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} \\
&= \lim_{h \to 0} \frac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} \\
&= \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}} \\
&= \frac{2}{2\sqrt{2x+1}} \\
&= \frac{1}{\sqrt{2x+1}}
\end{aligned}$$