Math 2215 - Calculus 1

Final Exam - 2017.12.11

Name: _____

Instructions: Please work out each problem in full detail. No points are given for a correct answer.

[5 pts] 1. State the Mean Value Theorem.

[5 pts] 2. State the Fundamental Theorem of Calculus.

For problems 3–7, compute each limit. If the limit does not exist, differentiate between $+\infty$, $-\infty$, or does not exist.

[6 pts] 3. $\lim_{x\to 0} \dfrac{\tan(3x)}{4x}$

[6 pts] 4. $\lim_{x\to 0} \sin\left(\dfrac{x}{\sin(x)}\right)$

[6 pts] 5. $\lim_{x\to -\infty} \dfrac{3x-4}{\sqrt{x^2-2x+1}}$

[6 pts] 6. $\lim_{x\to 4} \dfrac{x^2-x-12}{x-4}$

[6 pts] 7. $\lim_{x\to 4} \dfrac{x^2-x-14}{x-4}$

[8 pts] 8. Find values of a and b so that the following piecewise function is continuous everywhere.

$$f(x) = \begin{cases} \frac{2}{\pi}x - a, & x \le -\pi \\ b\cos(x) + 4, & -\pi < x < \pi \\ 3\sin\left(\frac{x}{2}\right) - 1, & x \ge \pi \end{cases}$$

9. For this problem, consider $f(x) = \dfrac{x+1}{x^2}$.

[2 pts] (a) What is the domain of $f(x)$?

[2 pts] (b) Locate the roots of $f(x)$

[4 pts] (c) Determine the locations of all vertical asymptotes

[3 pts] (d) Determine the locations of all horizontal/slant asymptotes

[4 pts] (e) Compute $f'(x)$. Be sure to simplify your answer

[3 pts] (f) Determine the critical points of $f(x)$ using $f'(x)$

[3 pts] (g) Compute the intervals of increase and decrease using $f'(x)$

[3 pts] (h) Classify the critical points as local maximums and minimums

[4 pts] (i) Compute $f''(x)$. Be sure to simplify your answer

[3 pts] (j) Determine intervals of concavity

[3 pts] (k) Locate all inflection points

[8 pts] (l) Sketch the graph of $f(x)$ using all of the information from parts (a)–(k)

[8 pts] 10. Show that the function $f(x) = x + \dfrac{x}{x^2+1}$ has *exactly* one root.

[6*2 pts] 11. Find the extreme values of $f(x) = 2x^3 - 9x^2 + 12x$ on the following intervals. (a) $[0, 3]$, (b) $[0, 2]$

For problems 12–15 compute the given derivative. You **do not** have to simplify your answer.

[6 pts] 12. $\dfrac{\mathrm{d}}{\mathrm{d}z}\left(\sin(z) + \cos(z)\right)^2$

[6 pts] 13. $\dfrac{\mathrm{d}}{\mathrm{d}w}\sqrt{\dfrac{\sin(w)+1}{\cos(w)-w}}$

[6 pts] 14. $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\sin(\cos(\tan(2\theta+1)))$

[6 pts] 15. $\dfrac{\mathrm{d}}{\mathrm{d}r}\displaystyle\int_{3r}^{2-r^3} \frac{1}{3}x^3 - \frac{1}{2}x^2 + x - 1\,\mathrm{d}x$

[8 pts] 16. Find **all** points on the graph $y^2 = x^3 - 3x + 1$ where the tangent line is horizontal.

[7 pts] 17. Compute the following integral: $\displaystyle\int \left(\sin(t) + \cos(t)\right)^2 \mathrm{d}t$

[7 pts] 18. Compute the following integral: $\displaystyle\int z\sqrt{3z+5}\,\mathrm{d}z$

[7 pts] 19. Compute the following integral: $\displaystyle\int \sqrt{z}(3z+5)\,\mathrm{d}z$

[8 pts] 20. Find the average value of the function $p(x) = x^2 - x + 2$ on the interval $[0, 4]$.

[8*2 pts] 21. Consider the finite region $\mathcal{R}$ bounded by the lines $y = -x+1$, $y = \dfrac{5}{3}x+1$, $y = \dfrac{1}{2}x - \dfrac{1}{2}$, and $y = -2x + 12$. Express the volume of the region $\mathcal{R}$ about the line $x = -2$ using **both** (a) $\mathrm{d}x$ and (b) $\mathrm{d}y$ integrals. You do not have to evaluate these integrals.

[8 pts] 22. Water pours into a conical tank of height 10 ft and radius 4 ft at a rate of 10 $\mathrm{ft}^3/\mathrm{min}$. How fast is the water level rising when it is 5 ft high? The volume of a cone is $V = \dfrac{\pi}{3}r^2h$.