

Math 2215 - Calculus 1

Quiz #20 - 2017.11.30

Solutions

Express the volume of the region bounded by $P(x) = 3x^2 - 5x - 2$ and $Q(x) = 2x^2 - 6x$ revolved around the line $x = -3$ as an integral. You do not have to evaluate the integral.

First, we set $P(x) = Q(x)$, which gives $x^2 + x - 2 = 0$, which can be factored as $(x - 1)(x + 2)$. So the interval on the x -axis over which we have an area to revolve is $[-2, 1]$. Since $P(x)$ and $Q(x)$ are both parabolas opening upwards, we need to figure out which one is on top. Since $x = 0$ is in the interval in question, we will use it as a test point: $P(0) = -2$ and $Q(0) = 0$. Thus $P(x) \leq Q(x)$ on the interval $[-2, 1]$. If we use slices Δx , then we would have a cylindrical shell whose radius is $r(x) = x - (-3) = x + 3$ and whose height is $Q(x) - P(x) = -x^2 - x + 2$. Setting up the integral, we now have

$$\mathcal{V} = \int_{-2}^1 2\pi (x + 3) (-x^2 - x + 2) dx$$