

# Math 2215 - Calculus 1

Quiz #21 - 2017.12.01

Solutions

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Express the volume of the region bounded by  $P(x) = 3x^2 - 5x - 2$  and  $Q(x) = 2x^2 - 6x$  revolved around the line  $y = 20$  as an integral. You do not have to evaluate the integral.

First, we set  $P(x) = Q(x)$ , which gives  $x^2 + x - 2 = 0$ , which can be factored as  $(x - 1)(x + 2)$ . So the interval on the  $x$ -axis over which we have an area to revolve is  $[-2, 1]$ . Since  $P(x)$  and  $Q(x)$  are both parabolas opening upwards, we need to figure out which one is on top. Since  $x = 0$  is in the interval in question, we will use it as a test point:  $P(0) = -2$  and  $Q(0) = 0$ . Thus  $P(x) \leq Q(x)$  on the interval  $[-2, 1]$ . If we use slices  $\Delta x$ , then we would have a washer whose outer radius is  $r_o(x) = 20 - P(x) = -3x^2 + 5x + 22$  and whose inner radius is  $r_i(x) = 20 - Q(x) = -2x^2 + 6x + 20$ . Setting up the integral, we now have

$$\mathcal{V} = \int_{-2}^1 \pi [(-3x^2 + 5x + 22)^2 - (-2x^2 + 6x + 20)^2] dx$$