

Math 2215 - Calculus 1

Quiz #23 - 2017.12.07

Solutions

For the function $f(x) = \frac{2x^2 + 1}{x^2 - 1}$, determine each of the following (if they exist):

(a) the domain

The domain is all real numbers except $x = \pm 1$, or $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$, or $\mathbb{R} - \{-1, 1\}$.

(b) roots

Since the numerator is always positive, there are no roots.

(c) y -intercept

The y -intercept is when $x = 0$, which gives a value of -1 .

(d) vertical asymptotes and the functions behaviour on each side of said asymptotes

There will be vertical asymptotes at $x = -1$ and $x = 1$. Since the numerator is always positive, we can look at the denominator solely to find the function's behaviour at the asymptotes. The denominator is only negative for $x \in (-1, 1)$, thus:

$$\lim_{x \rightarrow 1^+} f(x) = +\infty, \quad \lim_{x \rightarrow 1^-} f(x) = -\infty, \quad \lim_{x \rightarrow -1^+} f(x) = -\infty, \quad \lim_{x \rightarrow -1^-} f(x) = \infty$$

(e) horizontal asymptote

There is a horizontal asymptote since the degree of the denominator is equal to the degree of the numerator. So we take the ratio of the leading coefficients which gives a horizontal asymptote of $y = 2$.