Math 2283 - Introduction to Logic

Exam #3 - 2018.04.09

Name: _____

We have the following definitions:

R is *reflexive* $\overset{def}{\longleftrightarrow} \forall\, x\; xRx$

R is *irreflexive* $\overset{def}{\longleftrightarrow} \forall\, x\; \sim xRx$

R is *symmetrical* $\overset{def}{\longleftrightarrow} \forall\, x, y\; xRy \rightarrow yRx$

R is *asymmetrical* $\overset{def}{\longleftrightarrow} \forall\, x, y\; xRy \rightarrow \sim yRx$

R is *transitive* $\overset{def}{\longleftrightarrow} \forall\, x, y, z\; xRy \wedge yRz \rightarrow xRz$

R is *connected* $\overset{def}{\longleftrightarrow} \forall\, x, y\; \sim (x = y) \rightarrow (xRy \vee yRx)$

R is ***strongly connected*** $\overset{def}{\longleftrightarrow} \forall\, x, y\; xRy \vee yRx$

$xR \cup Sy \overset{def}{\longleftrightarrow} xRy \vee xSy$

$xR \cap Sy \overset{def}{\longleftrightarrow} xRy \wedge xSy$

$xR'y \overset{def}{\longleftrightarrow} \sim xRy$

$xR/Sy \overset{def}{\longleftrightarrow} \exists\, z\, (xRz \wedge zSy)$

$x\breve{R}y \overset{def}{\longleftrightarrow} yRx$

For the following definitions of the relations B, L, and W, you may assume that the x and y variables belong to the set of all recognized English words.

Define B to be the relation on words x and y which holds if the two words share the same first letter. As example, logic$B$like is a true sentence, (here x is 'logic', and y is 'like' in the xBy formulation we are used to using). Similarly: logic$B$dislike is a false sentence, since the words 'logic' and 'dislike' do not start with the same letter.

Define L to be the relation on words x and y which holds if the last letter of word x is the first letter of word y. For example, logic$L$confusion is a true sentence, (here x is 'logic' and y is 'confusion'), since 'logic' ends with the letter 'c' and 'confusion' starts with the letter 'c'. In a similar fashion, logic$L$fair is a false sentence.

Define W to be the relation on words x and y which holds if the last letter of word x is the first letter of word y, as well as the first letter of word x is the same as the last letter of word y. For example, logic$W$cool is a true sentence, (here x is 'logic' and y is 'cool'), since 'logic' starts with the letter 'l', ends with the letter 'c', and 'cool' starts with the letter 'c' and ends with the letter 'l'. In a similar fashion, logic$W$fail is a false sentence since the last letter of 'logic' is not the same as the first letter of 'fail'.

For problems 1–3, please explain your answers fully. If a relation does not have a specific property, give a counterexample to prove your point!

1. Determine which of the properties the relation B has: reflexive, irreflexive, symmetrical, asymmetrical, transitive, connected.

2. Determine which of the properties the relation L has: reflexive, irreflexive, symmetrical, asymmetrical, transitive, connected.

3. Determine which of the properties the relation W has: reflexive, irreflexive, symmetrical, asymmetrical, transitive, connected.

4. For each of the following, give an example of words x and y which make the sentences true:

(a) $xB \cup Ly$

(b) $xW \cap By$

(c) $x\breve{L}/By$

(d) $xB \cap \breve{L}y$

(e) $xB/(L/W)y$

5. Determine which of the following sentences are true, explain fully! You should not attempt actual logical proofs, but by explanations based on the definitions of B, L, and W.

(a) $W \subseteq B$

(b) $W \subseteq L$

(c) $L \cap \breve{L} = W$

6. Give a formal logical proof that if R' is strongly connected, then R is asymmetric. Note: the definition of strongly connected has been given on the first page of the exam, and the tautological statement $(\sim p \to q) \longleftrightarrow (p \vee q)$ might prove to be useful.