Math 2143 - Brief Calculus with Applications

Discussion Board Week 4 - Due 2018.07.01

So this week, we are going to do something a little clever, which is not in the book. We now know about logarithmic differentiation, and we also know all the wonderful properties of logarithms. Consider, for instance, if we wish to take the derivative of:

$$f(x) = \frac{(2x-3)(3x+5)}{4x^2+1}.$$

We can use the quotient rule and product rule (and chain rule), of course. However, consider the following strange idea. What if we take the derivative of the logarithm of $f(x)$:

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}.$$

Solving for $f'(x)$ gives

$$f'(x) = f(x) \cdot \frac{d}{dx}\ln(f(x)).$$

This may seem rather more complicated, but we use logarithm rules:

$$\begin{aligned} \ln(f(x)) &= \ln\left(\frac{(2x-3)(3x+5)}{4x^2+1}\right) \\ &= \ln\left((2x-3)(3x+5)\right) - \ln\left(4x^2+1\right) \\ &= \ln\left((2x-3)\right) + \ln\left(3x+5\right)) - \ln\left(4x^2+1\right). \end{aligned}$$

Now we can take the derivative of each of the above terms rather simply!

$$\frac{d}{dx}\ln(f(x)) = \frac{2}{2x-3} + \frac{3}{3x+5} - \frac{8x}{4x^2+1}.$$

We can now put it all together:

$$\frac{d}{dx}f(x) = \frac{(2x-3)(3x+5)}{4x^2+1} \cdot \left(\frac{2}{2x-3} + \frac{3}{3x+5} - \frac{8x}{4x^2+1}\right)$$

Use this new technique to compute the derivatives of the following rational functions.

1. $f(x) = \dfrac{(x+2)^3(x-1)}{(x+1)(x^2+3)}$
2. $f(x) = \dfrac{(x+2)(1-x)}{\sqrt{3x^2+1}}$
3. $f(x) = \dfrac{(x+2)(1-x)}{(2x+3)^2}$
4. $f(x) = \dfrac{(x-2)(1-x)(x+3)}{(x^2-4x)}$
5. $f(x) = \dfrac{(x-3)(1-x)}{(x^2-2)(x-4)^3}$
6. $f(x) = \dfrac{\sqrt{(x-3)}(1-x)}{(x-2)^2(x-4)}$
7. $f(x) = \dfrac{(x+3)^2(1-x)}{x(x-4)}$
8. $f(x) = \dfrac{(x+3)}{(x+1)^2(x-2)(x+4)}$
9. $f(x) = \dfrac{(x-3)\sqrt{(x+1)}}{\sqrt{(x-2)}(x+4)}$
10. $f(x) = \dfrac{(x-3)(x+1)^4}{(x-5)^2(x+4)^2}$
11. $f(x) = \dfrac{(x-3)(x+1)^4 x}{(x-1)^3\sqrt[3]{(x+2)}}$
11. $f(x) = \dfrac{(x-3)(x+1)^4}{(x-5)^2(x+4)^2}$