Math 4133 - Linear Algebra

Final Exam - 2018.04.19

Name: ______________________________

Assigned: 2018.04.19

Last Updated: 2018.04.25

Due: 2018.05.09 at 11:00 AM

Instructions: Work on this by yourself, the only person you may contact in any way to discuss or ask questions about this exam is Dr. Frinkle. For each problem, be sure to show all of your work and write every step down in a clear and concise manner. Please start each problem on a new sheet. When complete, staple all sheets in order to the cover page. You do not have to attach the remaining pages containing the actual questions if you do not so desire. Do NOT use decimal approximations on any problem for any reason.

Agreement: Please read the following statement and then write it at the bottom of the page before the signature line:

"*I hereby swear that all the work that appears on this written exam is completely my own, and I have not discussed any portion of this exam with any one else besides the instructor.*"

Printed Name: ______________________________

Signature: ______________________________

Date: ______________________________

Use the following system of equations to answer problems 1 – 5.

$$\begin{cases} 3x + 5y - 7z = 1 \\ 4x + 2y - 6z = 0 \\ 2x - 8y + 3z = -1 \end{cases}$$

1. Write the system of equations in $AX = B$ form.

2. Compute $\det(A)$.

3. Compute A^{-1}.

4. Compute $A^{-1}B$.

5. Write the solution to the system of equations using the previous problems.

Consider the set $\mathbf{K} = \{\langle 1, 2, 0, 1\rangle, \langle -1, 2, 1, 0\rangle\}$ for problems 6 – 10.

6. Compute the dimension of the vector space $\mathbb{S}$ spanned by $\mathbf{K}$.

7. Convert $\mathbf{K}$ to an orthonormal basis $\mathbf{B}$.

8. Compute a basis for $\mathbb{S}^\perp$, call it $\mathbf{K}^\perp$.

9. Convert $\mathbf{K}^\perp$ to an orthonormal basis $\mathbf{B}^\perp$.

10. Express the vector $\langle 1, -1, 1, -1\rangle$ as a linear combination of vectors from $\mathbf{B} \cup \mathbf{B}^\perp$.

Use the following system of equations to answer problems 11 – 14.

$$\begin{cases} 3x + 5y - 7z = 1 \\ 4x + 2y - 6z = -1 \end{cases}$$

11. Solve the homogeneous problem, and express the solution as a subspace of $\mathbb{R}^3$.

12. Solve the particular problem, and express the solution as a subspace translate of $\mathbb{R}^3$.

13. Find the equation of the plane through the origin perpendicular to your solution to problem 11 or 12.

14. Given the point $P(10, 5, -8)$ find the point on the plane closest to P, also give the distance.

Consider the linear map $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ defined by $T(\langle x, y, z\rangle) = \langle x + y + z, 0, x - y + 3z, 0\rangle$ for problems 15 – 19.

15. Compute the matrix A such that $T(\overrightarrow{v}) = A\overrightarrow{v}$.

16. Find a basis for $\mathrm{Im}(T)$. What is the dimension of $\mathrm{Im}(T)$?

17. Find a basis for $\text{Ker}(T)$. What is the dimension of $\text{Ker}(T)$?

18. Let

$$\mathbf{B}_3 = \{\langle 1,0,1\rangle, \langle 1,-1,0\rangle, \langle 1,1,1\rangle\}$$

and

$$\mathbf{B}_4 = \{\langle 1,0,1,0\rangle, \langle 1,0,-1,0\rangle, \langle 0,1,0,1\rangle, \langle 0,-1,0,1\rangle\}$$

be bases for $\mathbb{R}^3$ and $\mathbb{R}^4$, respectively. Draw a commutative diagram for the map T' which corresponds to the map T using the bases $\mathbf{B}_3$ and $\mathbf{B}_4$.

19. Using the diagram from problem 18, express the map T' as a matrix A' which is the product of three matrices corresponding to the diagram. You do NOT have to compute the inverse to any matrix.