Math 1513 - College Algebra

Exam #1 - 2018.09.21

Solutions

---

1. Find the equations of the lines passing through the point $(2, -4)$ which are (a) parallel and (b) perpendicular to the line $y = \frac{3}{2}x + 2$.

(a) For parallel lines, the slope remains the same, and we use point-slope form: $y + 4 = \frac{3}{2}(x - 2)$.

(b) For perpendicular lines, the slopes are negative reciprocals, and we use point-slope form: $y + 4 = -\frac{2}{3}(x - 2)$.

2. Perform the complex multiplication $(2 + \frac{1}{2}i)(3 - 4i)$, express your answer in standard complex form $a + bi$.

$$\begin{aligned}(2 + \frac{1}{2}i)(3 - 4i) &= 6 - 8i + \frac{3}{2}i - 2i^2 \\ &= 6 - 8i + \frac{3}{2}i + 2 \\ &= 8 - \frac{13}{2}i\end{aligned}$$

3. Perform the complex division $(2 + \frac{1}{2}i)\Big/(3 + 4i)$, express your answer in standard complex form $a + bi$.

$$\begin{aligned}\frac{2 + \frac{1}{2}i}{3 + 4i} &= \frac{2 + \frac{1}{2}i}{3 + 4i} \cdot \frac{3 - 4i}{3 - 4i} \\ &= \frac{(2 + \frac{1}{2}i)(3 - 4i)}{(3 + 4i)(3 - 4i)} \\ &= \frac{8 - \frac{13}{2}i}{3^2 + 4^2} \\ &= \frac{8}{25} - \frac{13}{50}i\end{aligned}$$

4. Solve the following equation: $\sqrt{2x + 1} - \sqrt{x} = 1$

First we move the $-\sqrt{x}$ to the right side of the equation:

$$\sqrt{2x + 1} = 1 + \sqrt{x}$$

Now we square both sides:

$$2x + 1 = 1 + 2\sqrt{x} + x$$

Since we still have $\sqrt{x}$ in the equation, we isolate it by itself on the right hand side.

$$x = 2\sqrt{x}$$

Squaring both sides again gives

$$x^2 = 4x$$

Moving everything to the left hand side and factoring gives

$$x(x - 4) = 0$$

Solutions to this equation are $x = 0$ and $x = 4$. Plugging both of these solutions into the original equation yield true statements, so they are both valid.

5. Solve the following equation: $2x + \frac{3}{x - 1} = -5$

First we multiply both sides by $x - 1$:

$$2x(x - 1) + 3 = -5(x - 1)$$

Multiplying everything out:

$$2x^2 - 2x + 3 = -5x + 5$$

Moving everything to the left hand side and combining like terms:

$$2x^2 + 3x - 2 = 0$$

At this point, we can factor or use the quadratic equation. In factored form, we have

$$(x + 2)(2x - 1) = 0$$

And thus our solutions are $x = -2$ and $x = \frac{1}{2}$. Plugging both of these back into our original equation validates both.

6. Solving the following inequality, express your answer in interval notation. $1 < |4x - 1| < 2$

Here we have to solve two inequalities, first: $1 < 4x - 1 < 2$ and the second: $-2 < 4x - 1 < -1$. The final answer will be the union of the two inequalities just given. Solving for x in each gives

$$\frac{1}{2} < x < \frac{3}{4} \quad \text{or} \quad -\frac{1}{4} < x < 0$$

In interval notation, the solutions are $(\frac{1}{2}, \frac{3}{4})$ and $(-\frac{1}{4}, 0)$. Thus, the final answer is

$$\left(-\frac{1}{4}, 0\right) \cup \left(\frac{1}{2}, \frac{3}{4}\right)$$

7. Determine the domain of the following function: $f(x) = \sqrt{\dfrac{2x + 1}{3x - 2}}$

We need the argument of the square root to be non-negative, i.e. $\frac{2x+1}{3x-2} \geq 0$. This means one of two things: $2x + 1 \geq 0$ and $3x - 2 > 0$, or $2x + 1 \leq 0$ and $3x - 2 < 0$.

The first set $2x + 1 \geq 0$ and $3x - 2 > 0$, solving for x in each yields $x \geq -\frac{1}{2}$ and $x > \frac{2}{3}$. The solution to both inequalities is the intersection of the two solutions, which is $x > \frac{2}{3}$.

The second set $2x + 1 \leq 0$ and $3x - 2 < 0$, solving for x in each yields $x \leq -\frac{1}{2}$ and $x < \frac{2}{3}$. The solution to both inequalities is the intersection of the two solutions, which is $x \leq -\frac{1}{2}$.

Finally, we take the union of the above two solution, which yields in interval notation:

$$\left(-\infty, -\frac{1}{2}\right] \cup \left(\frac{2}{3}, \infty\right)$$

8. For each of the following functions, determine if they are (I) even, (II) odd, or (III) neither.

(a) $f(x) = 2\sqrt{x^2 + 1} - 1$

$f(x)$ is even:

$$\begin{aligned} f(-x) &= 2\sqrt{(-x)^2 + 1} - 1 \\ &= 2\sqrt{x^2 + 1} - 1 \\ &= f(x) \end{aligned}$$

(b) $g(x) = 2\sqrt{x^2 + 1} - x^3 + 1$

$g(x)$ is neither:

$$\begin{aligned} g(-x) &= 2\sqrt{(-x)^2 + 1} - (-x)^3 + 1 \\ &= 2\sqrt{x^2 + 1} + x^3 + 1 \\ &\neq g(x) \\ &\neq -g(x) \end{aligned}$$

(c) $h(x) = 2x\sqrt{x^2+1} - x^3 + 3x$

$h(x)$ is odd:

$$\begin{aligned} h(-x) &= 2(-x)\sqrt{(-x)^2+1} - (-x)^3 + 3(-x) \\ &= -2x\sqrt{x^2+1} + x^3 - 3x \\ &= -(2x\sqrt{x^2+1} - x^3 + 3x) \\ &= -h(x) \end{aligned}$$

(d) $k(x) = 2x\sqrt{x^2+1} - x^2 + 1$

$k(x)$ is neither:

$$\begin{aligned} k(-x) &= 2(-x)\sqrt{(-x)^2+1} - (-x)^2 + 1 \\ &= -2x\sqrt{x^2+1} - x^2 + 1 \\ &\neq k(x) \\ &\neq -k(x) \end{aligned}$$

9. Solve the following equation: $-2(x^2+3) = -2x^2+6$

Distributing the parentheses on the left hand side gives $-2x^2 - 6 = -2x^2 + 6$. Cancelling the $-2x^2$ terms on each side, we end up with $-6 = 6$, which is a false statement, therefore there is no solution.

10. Solve the following equation: $-2(x^2+3) = -2x^2-6$

Distributing the parentheses on the left hand side gives $-2x^2 - 6 = -2x^2 - 6$. Since the left side equals the right side regardless of what value of x we use, the solution is all real numbers!