Math 1513 - College Algebra

Exam #3 - 2018.12.03

Solutions

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1. Solve the following equation for x: $e^x e^2 = \dfrac{e^4}{e^{x+1}}$.

$e^x e^2 = \dfrac{e^4}{e^{x+1}}$	original equation
$e^{x+2} = e^{4-(x+1)}$	product/quotient properties
$e^{x+2} = e^{3-x}$	simplify exponents
$x + 2 = 3 - x$	common base, relate exponents
$2x = 1$	simplify
$x = \dfrac{1}{2}$	solve for x

2. Solve the following equation for z: $\log_5(z-2) + \log_5(2z-3) = 2\log_5(z)$.

$\log_5(z-2) + \log_5(2z-3) = 2\log_5(z)$	original equation
$\log_5((z-2)(2z-3)) = \log_5(z^2)$	product and power properties
$5^{\log_5((z-2)(2z-3))} = 5^{\log_5(z^2)}$	base raised to both sides
$(z-2)(2z-3) = z^2$	inverse property
$2z^2 - 7z + 6 = z^2$	FOIL
$z^2 - 7z + 6 = 0$	simplify
$z - 6 = 0 \longrightarrow z = 6$	set first factor to zero and solve
$z - 1 = 0 \longrightarrow z = 1$	set second factor to zero and solve

Plugging both of these solutions into the original equation, we see that $z = 1$ is *not* a solution, only $z = 6$ is.

3. The following is the graph of an invertible function $f(x)$ whose domain is $(-\infty, 2) \cup (2, \infty)$ and whose range is $(-\infty, 3/4) \cup (3/4, \infty)$. On the same graph, sketch the inverse function, along with all possible vertical/horizontal asymptotes.

The horizontal asymptote of $y = 3/4$ (red dashed horizontal line) becomes a vertical asymptote of $x = 3/4$ (light blue dot-dashed vertical line), and the vertical asymptote at $x = 2$ (red dashed vertical line) becomes a horizontal asymptote at $y = 2$ (light blue dot-dashed horizontal line). Then the graph of $f(x)$ (solid blue) is reflected about the line $y = x$ (black dotted line).

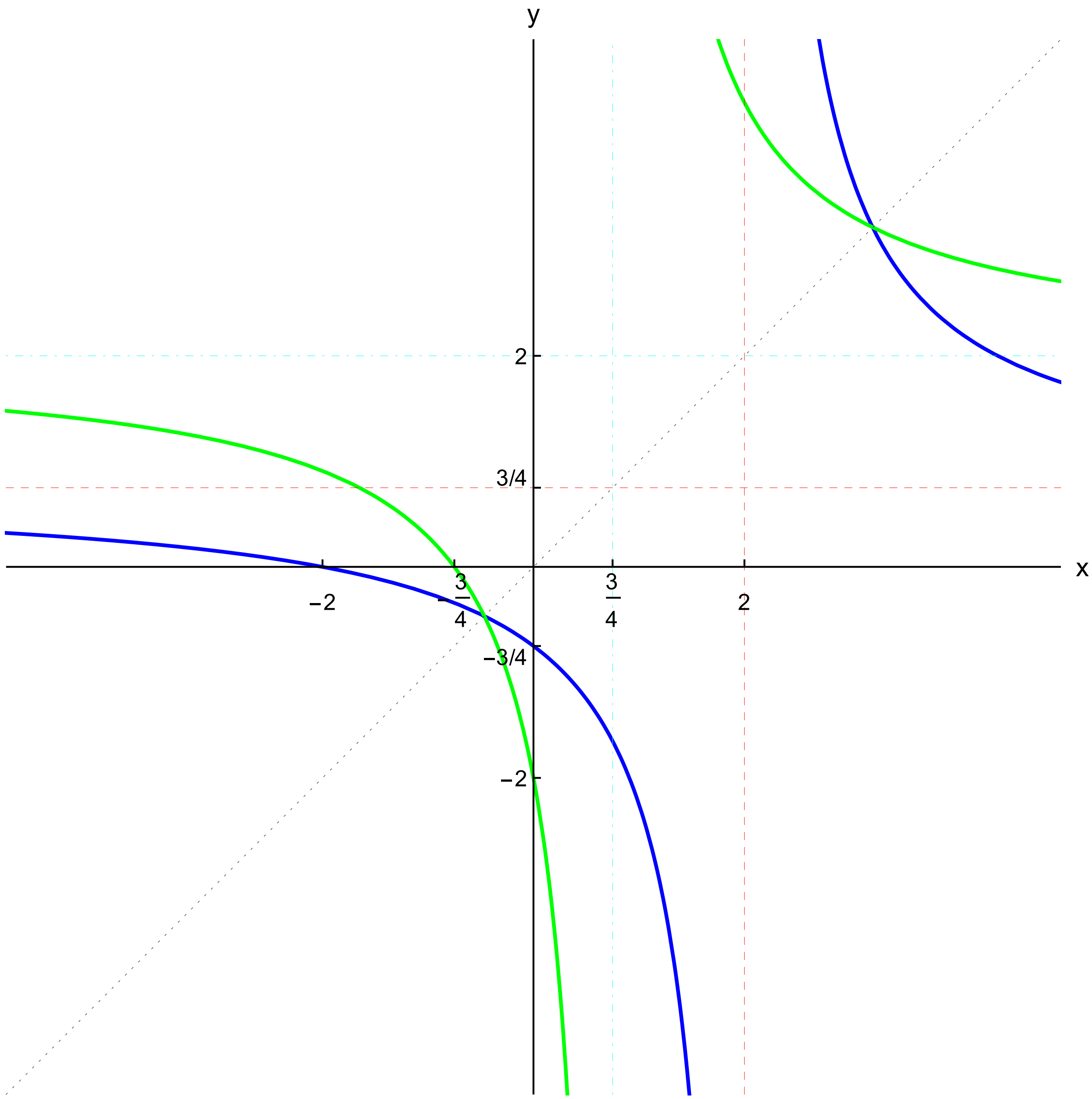

FIGURE 1. Graph of the invertible function $f(x)$ (blue) along with its inverse (green).

4. If $f(x) = \dfrac{3x+6}{4x-8}$, find the inverse function to $f(x)$. (Note, $f(x)$ does pass the horizontal line test, so is invertible.)

$$y = \frac{3x+6}{4x-8} \qquad \text{set } y = f(x)$$

$$x = \frac{3y+6}{4y-8} \qquad \text{swap variable } x \text{ and } y$$

$$x(4y-8) = 3y+6 \qquad \text{cross multiply by } 4y-8$$

$$4xy - 8x = 3y+6 \qquad \text{distribute to remove parentheses}$$

$$4xy - 3y = 8x+6 \qquad \text{isolate } y\text{'s on one side}$$

$$y(4x-3) = 8x+6 \qquad \text{factor out } y \text{ on LHS}$$

$$y = \frac{8x+6}{4x-3} \qquad \text{solve for } y$$

5. Solve the following system of linear equations:

$$\begin{cases} 2x + 4y &= 14, \\ -x + 2y &= 5 \end{cases}$$

There are many ways to solve this system however, if we take $2 \times$ row 2 $+$row 1 $\rightarrow$ row 2 we get

$$\begin{cases} 2x + 4y &= 14, \\ 8y &= 24 \end{cases}$$

Solving for y in the second equation gives $y = 3$. Then substituting $y = 3$ into either equation from the original system gives $x = 1$. Thus, our solution is $(x, y) = (1, 3)$.

6. Solve the following system of linear equations:

$$\begin{cases} x - y - z &= 0, \\ 2x + y - 3z &= -5, \\ -x + 3y + 2z &= 0 \end{cases}$$

We first eliminate the x variable from the second and third equations. To eliminate x from the second equation, we take row 2 $- 2 \times$ row 1 $\rightarrow$ row 2. To eliminate x from the third equation, we add rows 1 and 3 together and place the result in row 3, i.e. row 1 $+$ row 3 $\rightarrow$ row 3. This gives us

$$\begin{cases} x - y - z &= 0, \\ 3y - z &= -5, \\ 2y + z &= 0 \end{cases}$$

Adding the last two rows together gives $5y = -5$, or $y = -1$. Substituting this into $2y + z = 0$ gives $z = 2$, and then substituting $y = -1$ and $z = 2$ into the first equation gives $x + 1 - 2 = 0$ or $x = 1$. So our solution is $(x, y, z) = (1, -1, 2)$.

7. Solve the following system of nonlinear equations:

$$\begin{cases} 2x^2 + y^2 &= 24, \\ x^2 - y^2 &= -12 \end{cases}$$

Adding the two equations together gives $3x^2 = 12$ or $x^2 = 4$. Solving for x gives $x = \pm 2$, so we have two possible x-values for solutions. Plugging in $x = 2$ into the second equation gives $4 - y^2 = -12$. Solving for y gives $y^2 = 16$ or $y = \pm 4$. Similarly, plugging $x = -2$ into the second equation gives $4 - y^2 = -12$. Solving for y gives $y^2 = 16$ or $y = \pm 4$. Thus, we have four solutions: $(x, y) = (2, 4), (2, -4), (-2, 4), (-2, -4)$. Each of these four solutions do indeed satisfy the original system of equations.

8. Solve the following system of linear inequalities by graphing the solution region. *Be sure to clearly define your region.*

$$\begin{cases} y & < -2x + 6, \\ y & > -\frac{1}{3}x + 1, \\ y & < \frac{1}{2}x + 1 \end{cases}$$

We first begin by drawing each line in the xy-plane, using dashed lines:

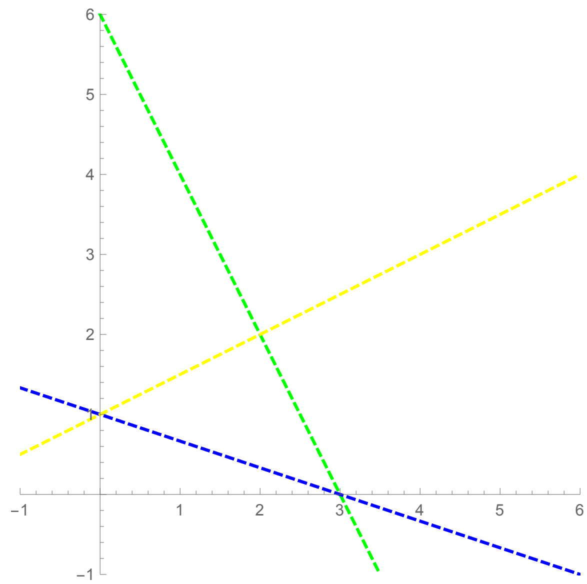

FIGURE 2. Graph of the lines from the system of inequalities.

Now we shade each region for which the inequality holds, we can test the point $(0, 0)$ for each inequality.

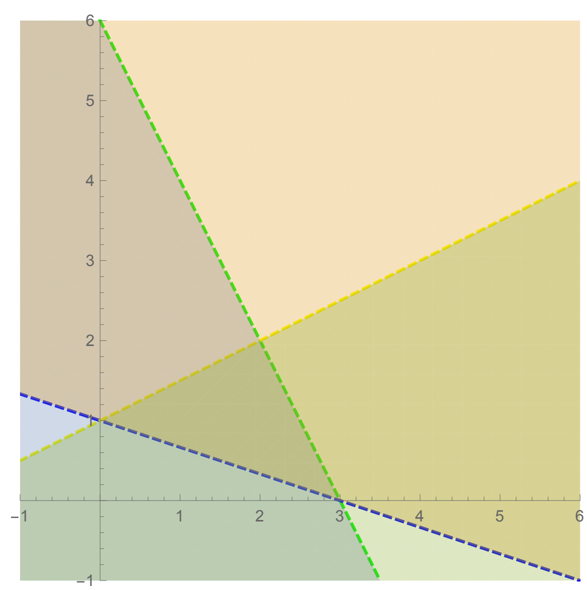

FIGURE 3. Graph of the lines from the system of inequalities and corresponding shaded regions for each inequality.

Lastly, we look at the region where all three shaded regions intersect. This happens to be the triangle formed by the points $(0,1)$, $(2,2)$ and $(3,0)$.

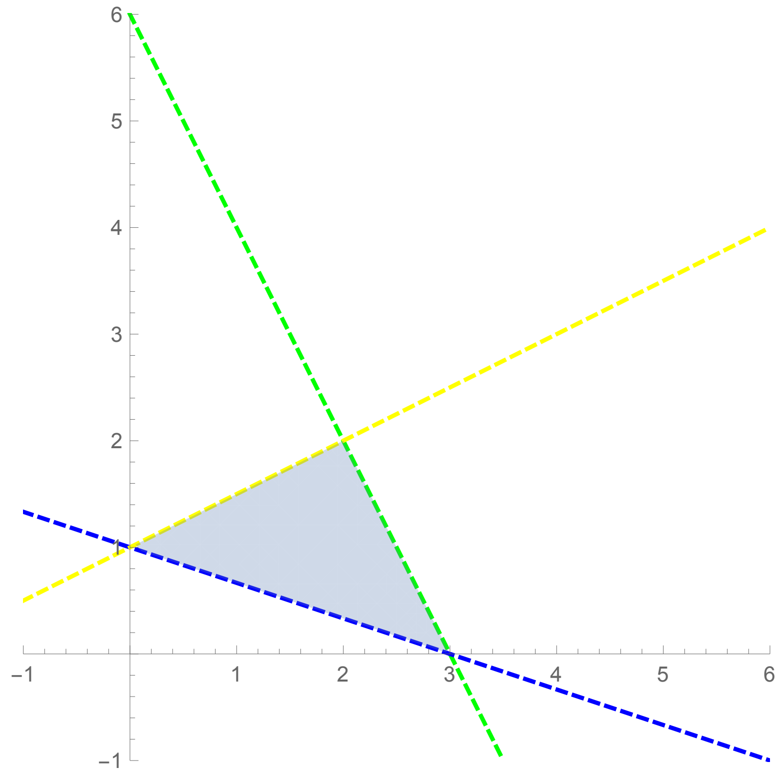

FIGURE 4. Region which satisfies all three inequalities is a triangle bounded by the points $(0,1)$, $(2,2)$ and $(3,0)$.