Math 1613 - Trigonometry

Exam #1 - 2018.09.12

Solutions

1. Find the values of the angles α, β, and γ using the image below:

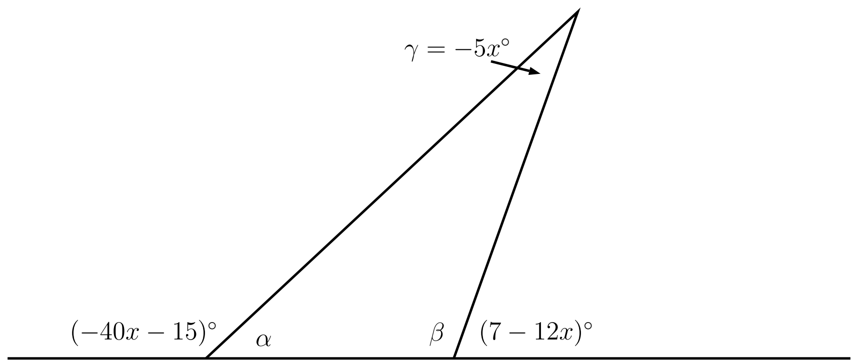

FIGURE 1. Excellent picture to go along with problem 1

The sum of interior angles adds up to 180°, thus $\alpha+\beta+\gamma = 180^\circ$. Furthermore, notice that $\alpha = 180^\circ - (-40x-15)^\circ$ and $\beta = 180^\circ - (7-12x)^\circ$. Thus

$$180^\circ - (-40x-15)^\circ + 180^\circ - (7-12x)^\circ - 5x^\circ = 180^\circ$$

Simplifying the above equation gives $188 + 47x = 0$, or $x = -4$. This gives $\gamma = 20^\circ$, $\beta = 125^\circ$, and $\alpha = 35^\circ$.

2. Find the values of all 6 trigonometric functions for the angle between the positive x-axis and the line segment starting at the origin and terminating at the point $(-2, 2\sqrt{3})$.

Here $x = -2$ and $y = 2\sqrt{3}$, so $r^2 = 4 + 12 = 16$, or $r = 4$. Now that we have x, y, and r, we can write down all the values of the 6 trig functions:

$$\sin(\theta) = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \qquad \cos(\theta) = -\frac{2}{4} = -\frac{1}{2} \qquad \tan(\theta) = -\frac{2\sqrt{3}}{2} = -\sqrt{3}$$

$$\csc(\theta) = \frac{2}{\sqrt{3}} \qquad \sec(\theta) = -2 \qquad \cot(\theta) = -\frac{1}{\sqrt{3}}$$

3. Fill out the following table completely:

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin(\theta)$	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$
$\cos(\theta)$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$	$-\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{2}{4}}$	$-\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{4}{4}}$

4. Find $\sec(\theta)$ given that $\tan(\theta) = \dfrac{\sqrt{7}}{3}$ and θ is in quadrant III.

First, since $\tan(\theta) = \dfrac{y}{x}$, and we are in quadrant III, we have that $y = -\sqrt{7}$ and $x = -3$. This gives $r = \sqrt{9+7} = 4$. Thus, $\sec(\theta) = \dfrac{r}{x} = -\dfrac{4}{3}$.

5. Find $\csc(\theta)$ given that $\cot(\theta) = -\dfrac{1}{2}$ and θ is in quadrant IV.

First, since $\cot(\theta) = \dfrac{x}{y}$, and we are in quadrant IV, we have that $x = 1$ and $y = -2$. This gives $r = \sqrt{1+4} = \sqrt{5}$. Thus, $\csc(\theta) = \dfrac{r}{y} = -\dfrac{\sqrt{5}}{2}$.

6. Find an angle α belonging to quadrants I or II such that the following equation holds:

$$\cos(2\alpha + 50^\circ) = \sin(2\alpha - 20^\circ)$$

Using the identity $\cos(\theta) = \sin(90^\circ - \theta)$, we get that

$$90^\circ - (2\alpha + 50^\circ) = 2\alpha - 20^\circ$$

Solving for α gives $\alpha = 15^\circ$.

7. Determine which is larger, $\sin(46^\circ)$ or $\cos(46^\circ)$.

Note that $\sin(45^\circ) = \cos(45^\circ)$, and as we increase the angle towards 90°, cos decreases, and sin increases, thus $\sin(46^\circ) > \cos(46^\circ)$.

8. Evaluate $\cot^2(135^\circ) + \tan^4(60^\circ) - \sin^4(180^\circ)$.

Notice that $\cot(135^\circ) = -1$, $\tan(60^\circ) = \sqrt{3}$, and $\sin(180^\circ)) = 0$. So

$$\cot^2(135^\circ) + \tan^4(60^\circ) - \sin^4(180^\circ) = (-1)^2 + \sqrt{3}^4 - 0^4 = 10.$$

9. Convert $23^\circ 15'$ to decimal degree.

Since $15'$ is a quarter of a full degree, we have that $23^\circ 15' = 23.25^\circ$.

10. Perform the following calculation: $110^\circ 25' - 92^\circ 43'$.

We start by rewriting a full degree of the first value as 60 minutes, then perform the calculation:

$$\begin{aligned} 110^\circ 25' - 92^\circ 43' &= 109^\circ 85' - 92^\circ 43' \\ &= 17^\circ 42' \end{aligned}$$

11. You drag your 6 ft tall favorite math professor outside, and measure his shadow, which happens to be 2 feet long. The math building's shadow, at this time, happens to be 11 feet long. How tall is the Math building?

This is a similar triangles problem. If we let h be the height of the Math building, then $\dfrac{6}{2} = \dfrac{h}{11}$. Solving for h gives $h = 33$ feet.

12. Use the trigonometric identity $\sin^2(\theta)+\cos^2(\theta) = 1$ to derive two more trigonometric identities, the first involving $\tan^2(\theta)$, and the second involving $\cot^2(\theta)$.

If we start with $\sin^2(\theta) + \cos^2(\theta) = 1$ and divide everything by $\cos^2(\theta)$, we get

$$\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

and using trig identities, this becomes:

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

Similarly, if we divide the identity by $\sin^2(\theta)$ instead, we have

$$1 + \cot^2(\theta) = \csc^2(\theta)$$