Math 1613 - Trigonometry

Exam #2 - 2018.10.10

Solutions

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1. Fill out the following table completely:

$\theta°$	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
θ (rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π
$\sin(\theta)$	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$
$\cos(\theta)$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$	$-\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{2}{4}}$	$-\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{4}{4}}$

2. Convert 1355° to radian measure.

$$1355° \cdot \frac{2\pi \text{ rad}}{360°} = \frac{89}{12}\pi$$

3. Convert $-\frac{17}{3}\pi$ radians to degree measure.

$$-\frac{17}{3}\pi \cdot \frac{360°}{2\pi \text{ rad}} = -1020°$$

4. Find the measure of the central angle θ corresponding to a circle of radius 12 inches whose arc has length 63 inches.

Here we use the formula $s = r\theta$, with $s = 63$ and $r = 12$, both in inches, to get $\theta = \frac{s}{r} = \frac{63}{12}$.

5. Find the measure of the central angle θ corresponding to a circle of radius 12 inches and sector of area 96π square inches.

Here we use the formula $A = \frac{\theta}{2\pi}\pi r^2$, with $A = 96\pi$ and $r = 12$ and solving for θ gives

$$\theta = A\frac{2}{r^2} = 96\pi \cdot \frac{2}{12^2} = \frac{4}{3}\pi$$

6. The minute hand of a standard 12 hour wall clock is 7 inches long, the hour hand is 4 inches long.
(a) Determine the angular velocity (in rad/sec) of the tip of each hand.

So remember that $\omega = \frac{\theta}{t}$, and in this case, the minute hand goes through $\omega = \frac{2\pi}{60}\frac{\text{rad}}{\text{min}}$. Converting this to the desired units gives

$$\omega = \frac{2\pi}{60}\frac{\text{rad}}{\text{min}} \cdot \frac{1 \text{ min}}{60\text{sec}} = \frac{2\pi}{60} \cdot \frac{1}{60}\frac{\text{rad}}{\text{sec}} = \frac{\pi}{1800}\frac{\text{rad}}{\text{sec}}$$

For the hour hand, we have that it goes through 1 revolution per 12 hours, thus

$$\omega = \frac{2\pi}{12}\frac{\text{rad}}{\text{hours}} \cdot \frac{11 \text{ hour}}{3600\text{sec}} = \frac{2\pi}{12} \cdot \frac{1}{3600}\frac{\text{rad}}{\text{sec}} = \frac{\pi}{21600}\frac{\text{rad}}{\text{sec}}$$

(b) Compute the velocity (in ft/sec) of the tip of each hand.

The relation between velocity v and angular moment ω is $v = r\omega$. So for the minute had:

$$v = 7 \text{ in} \cdot \frac{\pi}{1800}\frac{\text{rad}}{\text{sec}} = \frac{7\pi}{1800}\frac{\text{in}}{\text{sec}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{7\pi}{21600}\frac{\text{ft}}{\text{sec}}$$

For the hour hand:

$$v = 4 \text{ in} \cdot \frac{\pi}{21600} \frac{\text{rad}}{\text{sec}} = \frac{4\pi}{21600} \frac{\text{in}}{\text{sec}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{\pi}{64800} \frac{\text{ft}}{\text{sec}}$$

7. Sketch the graph of the function $y = \frac{2}{3} \cos\left(\frac{4}{5}x + \frac{\pi}{2}\right) - 1$ over two periods.

Start with the principle period:

$$0 \leq \frac{4}{5}x + \frac{\pi}{2} \leq 2\pi \longrightarrow -\frac{5}{8}\pi \leq x \leq \frac{15}{8}\pi$$

The box will be shifted down 1 unit to be centered at $y = -1$, with amplitude $\frac{2}{3}$.

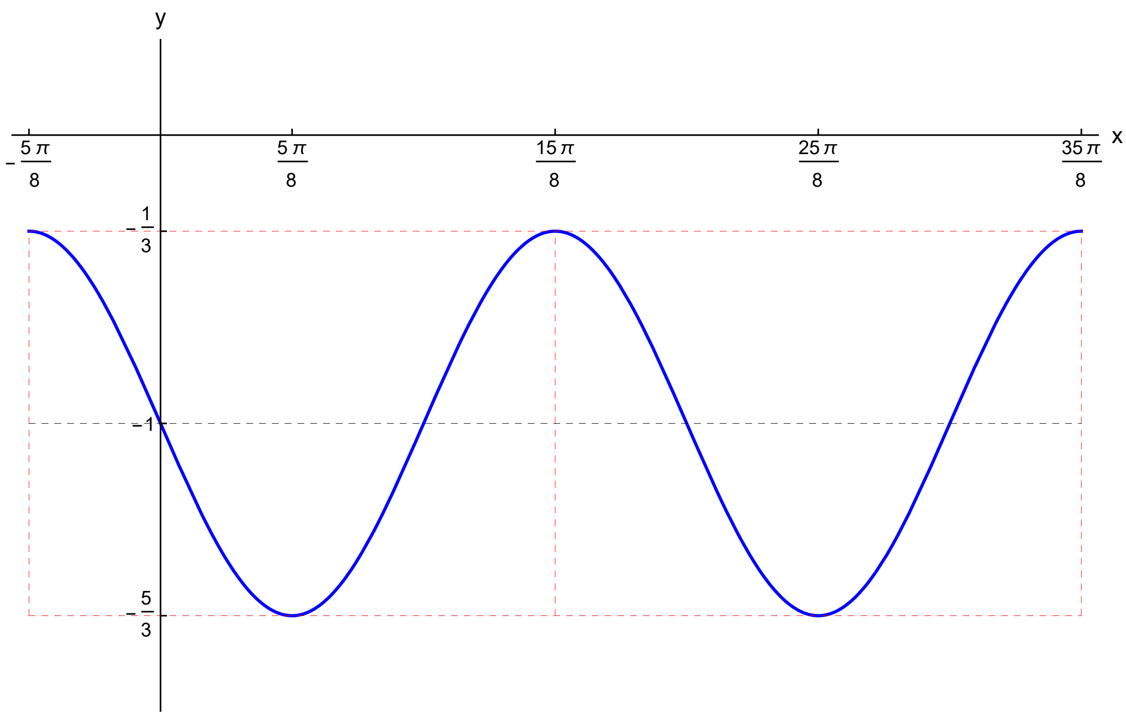

FIGURE 1. Graph of $y = \frac{2}{3} \cos\left(\frac{4}{5}x + \frac{\pi}{2}\right) - 1$

8. Sketch the graph of the function $y = \frac{3}{2}\sin\left(-\frac{1}{3}x + \frac{\pi}{6}\right) + \frac{1}{2}$ over two periods.

First, we pull a negative sign out of the argument of sine:

$$y = \frac{3}{2}\sin\left(-\frac{1}{3}x + \frac{\pi}{6}\right) + \frac{1}{2} = -\frac{3}{2}\sin\left(\frac{1}{3}x - \frac{\pi}{6}\right) + \frac{1}{2}$$

Start with the principle period:

$$0 \leq \frac{1}{3}x - \frac{\pi}{6} \leq 2\pi \longrightarrow \frac{\pi}{2} \leq x \leq \frac{13}{2}\pi$$

The box will be shifted up by $\frac{1}{2}$ units to be centered and reflected across the line $y = \frac{1}{2}$, with amplitude $\frac{3}{2}$.

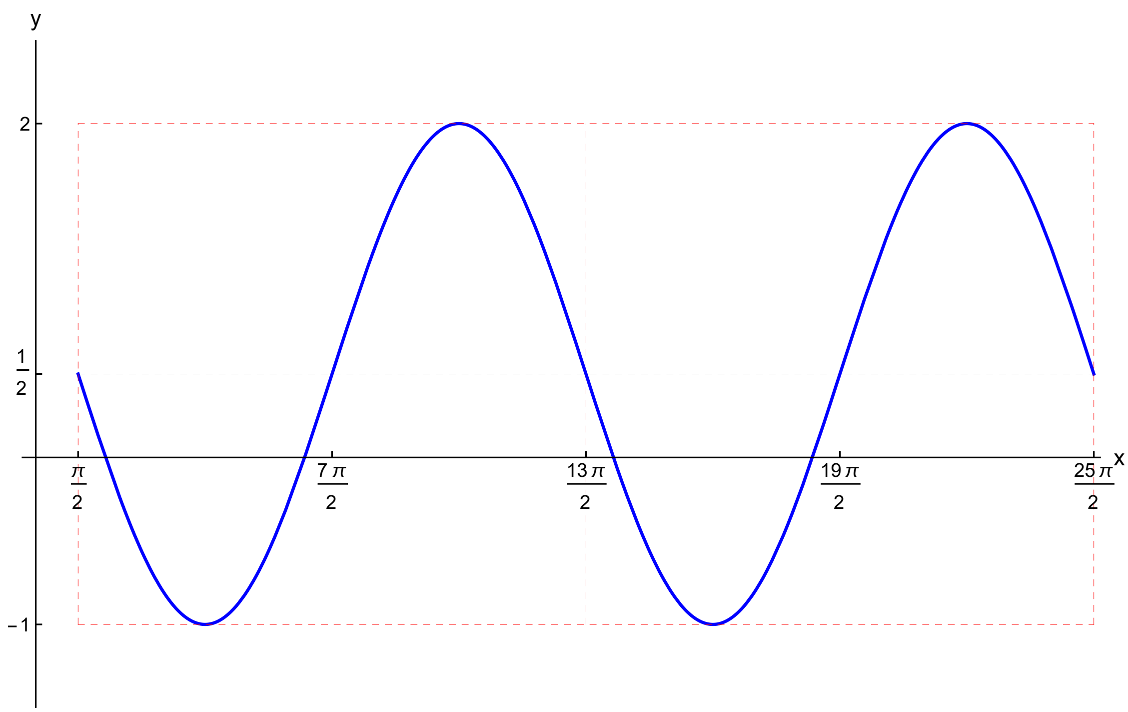

Figure 2. Graph of $y = \frac{3}{2}\sin\left(-\frac{1}{3}x + \frac{\pi}{6}\right) + \frac{1}{2}$

9. Sketch the graph of the function $y = \dfrac{1}{2}\tan\left(3x - \dfrac{\pi}{4}\right) + 4$ over two periods.
Start with the principle period:

$$-\frac{\pi}{2} \leq 3x - \frac{\pi}{4} \leq \frac{\pi}{2} \longrightarrow -\frac{\pi}{12} \leq x \leq \frac{\pi}{4}$$

The graph is compressed by a factor of 2 vertically and shifted up 4 units.

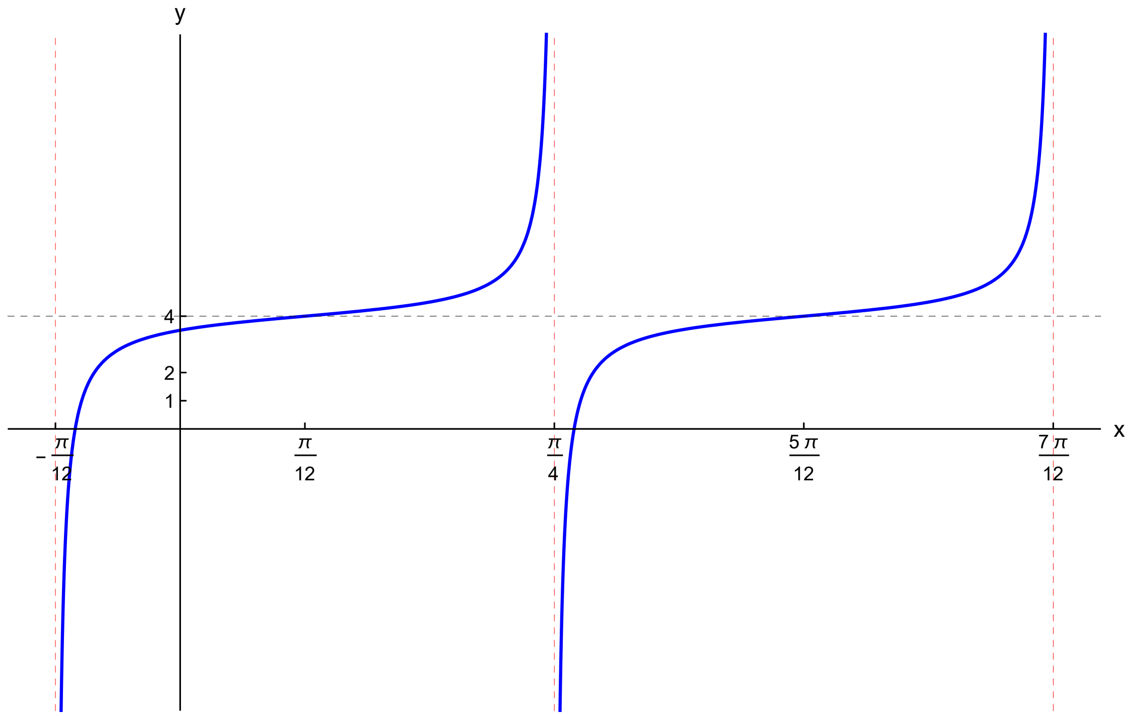

FIGURE 3. Graph of $y = \frac{1}{2}\tan\left(3x - \frac{\pi}{4}\right) + 4$

10. Sketch the graph of the function $y = -\frac{1}{2}\cot\left(3x + \frac{\pi}{4}\right) + 4$ over two periods.
Start with the principle period:

$$0 \leq 3x + \frac{\pi}{4} \leq \pi \longrightarrow -\frac{\pi}{12} \leq x \leq \frac{\pi}{4}$$

So the graph of this function, over the principle period, is on the interval $\left(-\frac{\pi}{12}, \frac{\pi}{4}\right)$, but flipped about the vertical midpoint due to the $-$ out front. Since the principle period is the same of that from the previous problem, and the graphs of tangent and cotangent are the same when one is flipped about the midpoint of the interval, we get that the graph of the current function is the same as that from problem 9.

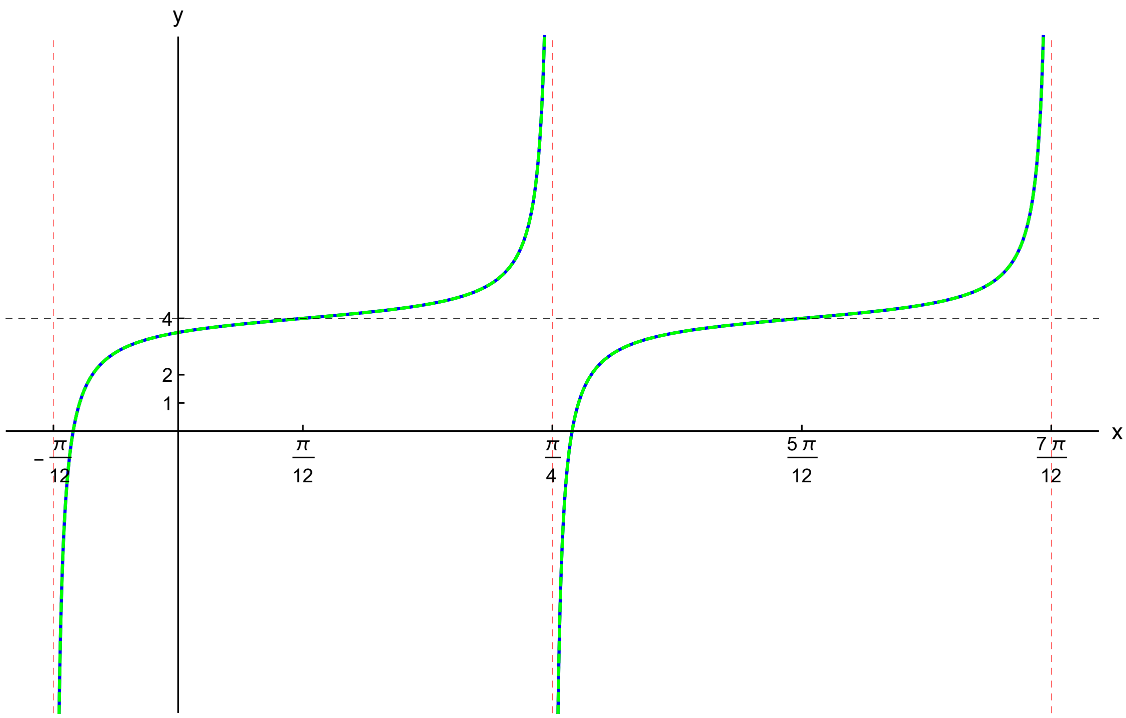

FIGURE 4. Graph of $y = -\frac{1}{2}\cot\left(3x + \frac{\pi}{4}\right) + 4$ (green) and $y = \frac{1}{2}\tan\left(3x - \frac{\pi}{4}\right) + 4$ (blue dots)

11. Sketch the graph of the function $y = \frac{2}{3}\sec\left(\frac{4}{5}x + \frac{\pi}{2}\right) - 1$ over two periods.

We simply reuse the information from problem 7 to graph the current function.

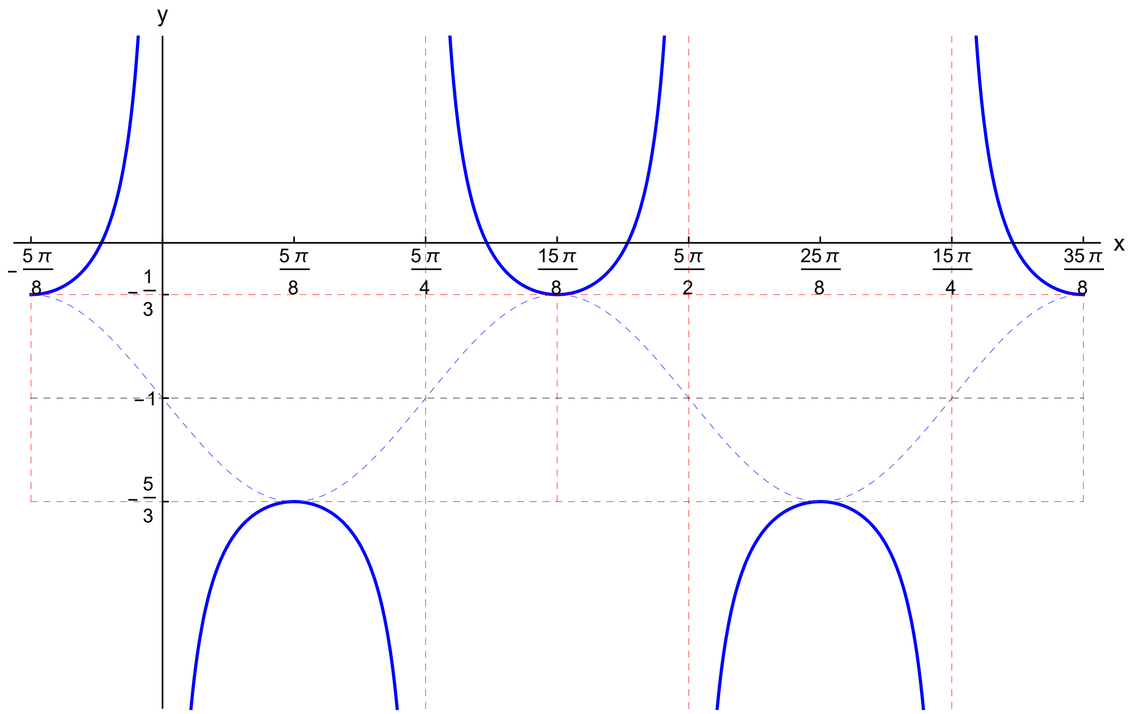

FIGURE 5. Graph of $y = \frac{2}{3}\sec\left(\frac{4}{5}x + \frac{\pi}{2}\right) - 1$

12. Sketch the graph of the function $y = \frac{3}{2}\csc\left(-\frac{1}{3}x + \frac{\pi}{6}\right) + \frac{1}{2}$ over two periods.

We simply reuse the information from problem 8 to graph the current function.

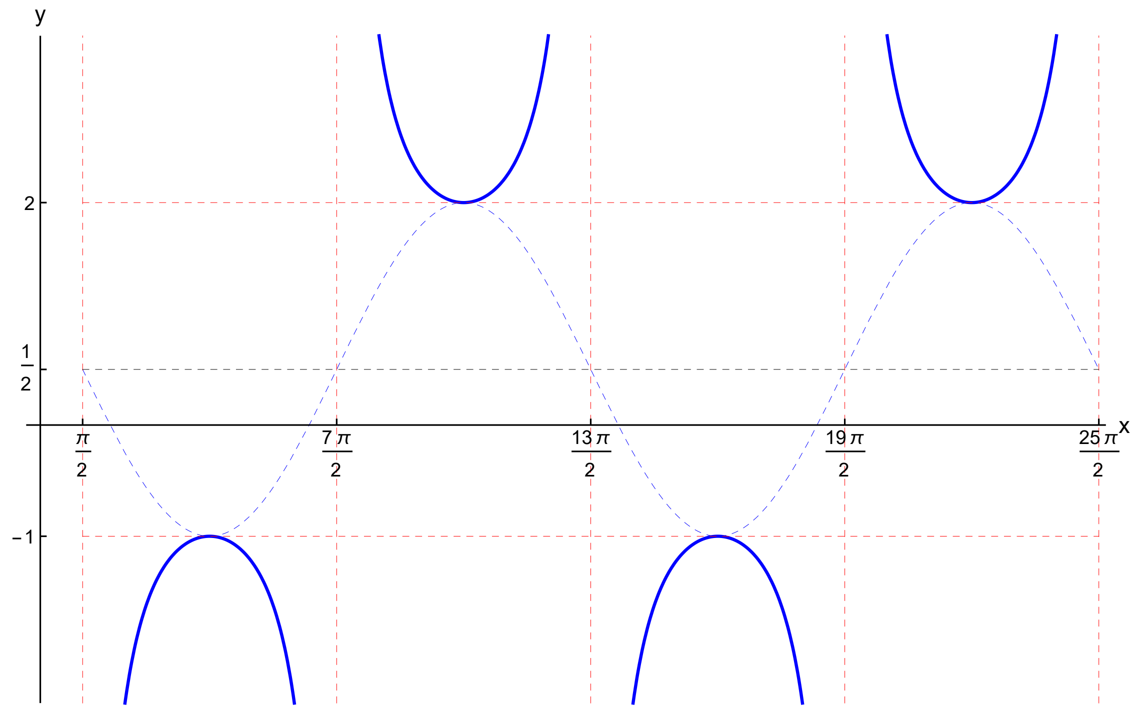

FIGURE 6. Graph of $y = \frac{2}{3}\sec\left(\frac{4}{5}x + \frac{\pi}{2}\right) - 1$

13. Find an equation of a function of the form $y = a\cos(bx + c) + d$ whose graph is given by the following figure:

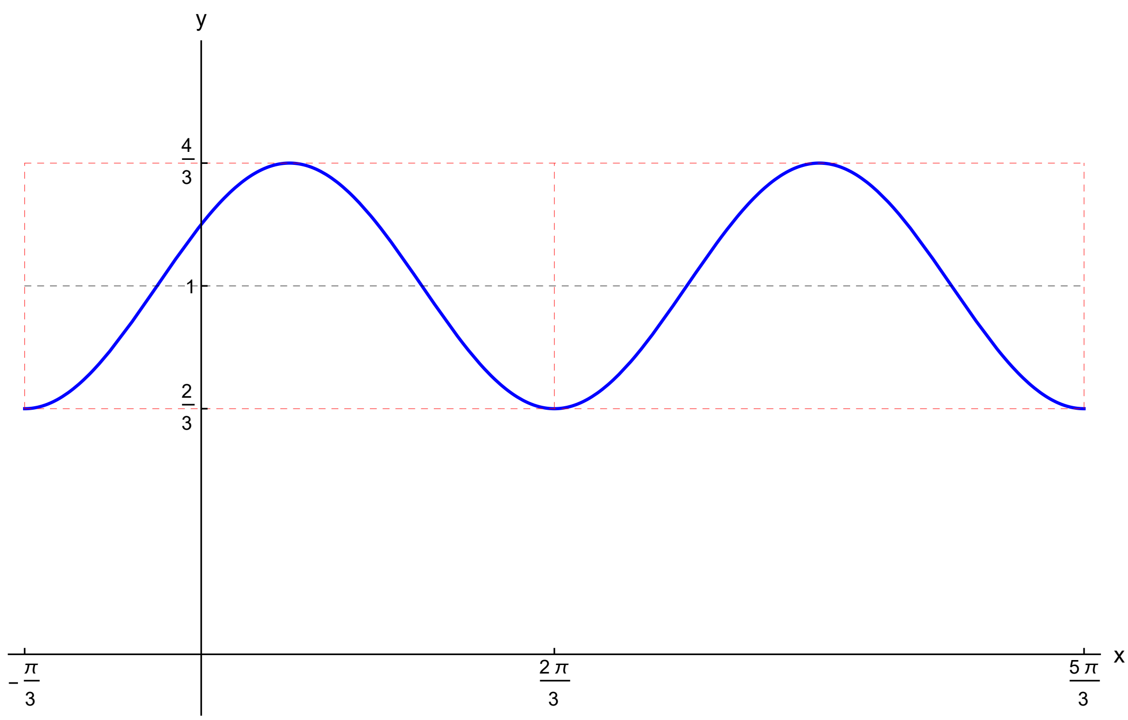

FIGURE 7. This is the graph of some cosine function, figure it out!

The amplitude is $\frac{1}{3}$, and since the cosine graph is flipped about the horizontal line of symmetry, we have that $a = -\frac{1}{3}$. Also, the vertical shift is $d = 1$. To find b and c, we simply solve the inequality:

$$-\frac{\pi}{3} \leq x \leq \frac{2}{3}\pi \longrightarrow 0 \leq x + \frac{\pi}{3} \leq \pi \to 0 \leq 2x + \frac{2}{3}\pi \leq 2\pi$$

Therefore, our function is $y = -\frac{1}{3}\cos\left(2x + \frac{2}{3}\pi\right) + 1$.