Math 1613 - Trigonometry

Exam #3 - 2018.11.09

Name: _____

Sum and Difference Identities:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1+\tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Cofunction Identities:

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta), \quad \sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta), \quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot(\theta)$$

Product-to-Sum and Sum-to-Product Identities:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$
$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$
$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$
$$\cos(A)\sin(B) = \frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$
$$\sin(A)+\sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A)-\sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A)+\cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A)-\cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Double-Angle Identities:

$$\cos(2A) = \cos^2(A)-\sin^2(A) = 1-2\sin^2(A) = 2\cos^2(A)-1$$
$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Half-Angle Identities:

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}, \quad \cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$
$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}}, \quad \tan\left(\frac{A}{2}\right) = \frac{\sin(A)}{1+\cos(A)}, \quad \tan\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{\sin(A)}$$

1. [10 pts] Fill out the following table completely:

$\theta°$	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
θ (rad)									
$\sin(\theta)$									
$\cos(\theta)$									

For each of the following equations find exact solutions on the interval $[0, 2\pi)$.

2. [15 pts] $\csc(x) + \cot(x) = 1$

3. [15 pts] $\cos^2(\theta) = \cos(\theta) + \sin^2(\theta)$

Verify each of the following trigonometric identities.

4. [15 pts] $\dfrac{\sin(2t)+\sin(4t)}{\cos(2t)-\cos(4t)} = \cot(t)$

5. [15 pts] $\sin(x)\sin(y)\sin(z) = \dfrac{1}{4}\left[\sin(x+y-z)+\sin(y+z-x)+\sin(z+x-y)-\sin(x+y+z)\right]$

6. [15 pts] Write $\sin(2\cot^{-1}(x))$ as an algebraic expression only, free of trigonometric or inverse trigonometric functions.

7. [15 pts] Solve the following equation over the interval $[0, 2\pi)$: $\sin\left(\frac{5}{2}x\right) = -\frac{\sqrt{3}}{2}$.