Math 1613 - Trigonometry

Final Exam - 2018.12.10

Name: ______________________________

Sum and Difference Identities:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1+\tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Cofunction Identities:

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta), \quad \sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta), \quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot(\theta)$$

Product-to-Sum and Sum-to-Product Identities:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$
$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$
$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$
$$\cos(A)\sin(B) = \frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$
$$\sin(A)+\sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A)-\sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A)+\cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A)-\cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Double-Angle Identities:

$$\cos(2A) = \cos^2(A)-\sin^2(A) = 1-2\sin^2(A) = 2\cos^2(A)-1$$
$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Half-Angle Identities:

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}, \quad \cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$
$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}}, \quad \tan\left(\frac{A}{2}\right) = \frac{\sin(A)}{1+\cos(A)}, \quad \tan\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{\sin(A)}$$

1. [10 pts] Fill out the following table completely:

$\theta°$	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
θ (rad)									
$\sin(\theta)$									
$\cos(\theta)$									

2. Convert $17°15'$ to decimal degrees.

3. Convert $29.45°$ to degrees, minutes, and seconds.

4. Convert 625° to radian measure.

5. Convert $\frac{3}{17}\pi$ radians to degree measure.

6. If the area $\mathcal{A}$ of the sector of a circle with central angle $\theta = \frac{1}{2}$ radians is 100 square units, what is the radius of the circle?

7. Sketch the graph of $f(x) = 2\sin\left(\frac{2}{3}x - \frac{\pi}{2}\right) - 2$ over two full periods.

8. Sketch the graph of $f(x) = \tan(3x + \pi) + 1$ over two full periods.

9. Sketch the graph of $f(x) = \dfrac{1}{2}\sec\left(2x - \dfrac{\pi}{4}\right) + 1$ over two full periods.

10. Prove the following trigonometric identity:

$$\frac{\tan^3(t) - \cot^3(t)}{\tan(t) - \cot(t)} = \sec^2(t) + \cot^2(t)$$

Hint: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

11. Prove the following trigonometric identity:

$$\sin(\theta) + \cos(\theta) + 1 = \frac{2\sin(\theta)\cos(\theta)}{\sin(\theta) + \cos(\theta) - 1}$$

12. Prove the following trigonometric identity:

$$\tan(A+B+C) = \frac{\tan(A)+\tan(B)+\tan(C)+\tan(A)\tan(B)\tan(C)}{1+\tan(A)\tan(B)+\tan(A)\tan(C)+\tan(B)\tan(C)}$$

13. Prove the following trigonometric identity:

$$\frac{\cos(5w) + \cos(w)}{\cos(w) - \cos(5w)} = \frac{\cot(2w)}{\tan(3w)}$$

14. Compute exactly $\tan\left(\frac{\pi}{8}\right)$.

15. Find all solutions for $0 \leq x < 2\pi$ to the equation: $\sin(2x - 1) = \frac{1}{2}$.

16. Find all solutions for $0 \leq z < 2\pi$ to the equation: $1 - \sin(z) = \cos(2z)$.

17. Write $\cos(2\tan^{-1}(x))$ as an algebraic expression only, free of trigonometric or inverse trigonometric functions.

18. A triangle has corners given by the points $(1,2)$, $(-1,1)$, and $(-3,4)$. Graph this triangle in the xy-plane and use the vector approach to compute the cosine of each angle in the triangle. Remember $\cos(\theta) = \dfrac{\overrightarrow{u} \cdot \overrightarrow{v}}{|\overrightarrow{u}|\,|\overrightarrow{v}|}$.

19. Given the vector $\overrightarrow{u} = \langle 2, 3, -1, 1, 4 \rangle$, find two nonzero vectors $\overrightarrow{v}$ and $\overrightarrow{w}$ which are perpendicular to $\overrightarrow{u}$ such that $\overrightarrow{v}$ and $\overrightarrow{w}$ do not lie on the same line.