Math 2283 - Honors Logic

Midterm - 2019.03.27

Name: ______________________________

Potentially Useful Information:

Tautological Sentences:

(1) $\sim (p \vee q) \longleftrightarrow \sim p \wedge \sim q$
(2) $\sim (p \wedge q) \longleftrightarrow \sim p \vee \sim q$
(3) $p \vee q \longleftrightarrow \sim (\sim p \wedge \sim q)$
(4) $p \wedge q \longleftrightarrow \sim (\sim p \vee \sim q)$
(5) $(p \to q) \longleftrightarrow \sim p \vee q$
(6) $(p \vee \sim p) \longleftrightarrow T$
(7) $(\sim p \vee p) \longleftrightarrow T$
(8a) $(p \wedge q) \longleftrightarrow (q \wedge p)$
(8b) $(p \vee q) \longleftrightarrow (q \vee p)$
(9a) $p \wedge T \longleftrightarrow p$ (Here T is any true sentence)
(9b) $T \wedge p \longleftrightarrow p$ (Here T is any true sentence)
(10) $p \vee F \longleftrightarrow p$ (Here F is any false sentence)
(11a) $p \wedge (q \vee r) \longleftrightarrow (p \wedge q) \vee (p \wedge r)$
(11b) $(p \vee q) \wedge r \longleftrightarrow (p \wedge r) \vee (q \wedge r)$
(12a) $p \vee (q \wedge r) \longleftrightarrow (p \vee q) \wedge (p \vee r)$
(12b) $(p \wedge q) \vee r \longleftrightarrow (p \vee r) \wedge (q \vee r)$

Class Properties:

$x \in K' \overset{def}{\longleftrightarrow} \sim x \in K$
$x \in K \cup L \overset{def}{\longleftrightarrow} x \in K \vee x \in L$
$x \in K \cap L \overset{def}{\longleftrightarrow} x \in K \wedge x \in L$

Relation Properties:

R is symmetric $\overset{def}{\longleftrightarrow} \forall x, y \in K \ xRy \to yRx$
R is reflexive $\overset{def}{\longleftrightarrow} \forall x \in K \ xRx$
R is transitive $\overset{def}{\longleftrightarrow} \forall x \in K \ xRy \wedge yRz \longrightarrow xRz$

Point distribution

Problem	Points Total	Points Earned
0(a)	10	
0(b)	10	
0(c)	10	
1(a)	25	
1(b)	25	
2	45	
3(a)	15	
3(b)	15	
4	45	
Total	200	

0. State the following as clearly as possible:

(a) The Rule of Detachment

(b) The Rule of Substitution

(c) The Rule of And, Joining Together

1. We know that the disjunction of two sentences can be written in terms of negation and conditional connectives, i.e. $(p \vee q) \longleftrightarrow (\sim p \rightarrow q)$. In a similar fashion, express (a) $p \wedge q$ and (b) $p \longleftrightarrow q$ using only $\sim$, $\rightarrow$ and parentheses (,).

2. A relation R is said to be **Co-Reflexive** in the class K if for any x and y in K, if xRy, then $x = y$. I.e.

$$R \text{ is } \textit{Co-Reflexive} \overset{def}{\longleftrightarrow} \forall x, y \in K \ xRy \rightarrow x = y.$$

A relation R is said to be **Antisymmetric** in the class K if for any x and y in K, if xRy and yRx, then $x = y$. In logical definition form:

$$R \text{ is } \textit{Antisymmetric} \overset{def}{\longleftrightarrow} \forall x, y \in K \ xRy \wedge yRx \rightarrow x = y.$$

Prove the following theorem of relations:

Theorem: If a relation R is symmetric and antisymmetric on a class K, then the relation R is co-reflexive on the class K.

Hint: Assume the hypothesis of the definition of R being co-reflexive, namely: xRy, and then, using the definitions of R being symmetric and antisymmetric, derive the conclusion to the definition of R being co-reflexive: $x = y$.

3. Express the properties of (a) co-reflexivity and (b) antisymmetry of a relation R using relations. As a reminder, the property of asymmetry for a relation R can be expressed by $R \subseteq (\breve{R})'$. Also, I was the relation of being *identical*, and D for *different*.

4. Define the class operation Δ on two classes K and L as follows:

$$x \in K\Delta L \overset{def}{\longleftrightarrow} (x \in K \vee x \in L) \wedge \sim (x \in K \wedge x \in L)$$

In a sentence, elements of the class $K\Delta L$ belong to either K or L but not in the intersection of K and L.

Furthermore, define the class operation $-$ on two classes K and L as follows:

$$x \in K - L \overset{def}{\longleftrightarrow} x \in K \wedge \sim x \in L$$

In a sentence, elements of the class $K - L$ belong to K and do not belong to L.

Prove the following class based theorem:

Theorem: $K\Delta L = (K - L) \cup (L - K)$