

# Math 3283 - Foundations of Mathematics

Exam 1 - 2019.02.11

Name: \_\_\_\_\_

1. Construct a the truth table for the sentence  $((p \vee q) \rightarrow r) \leftrightarrow ((p \rightarrow r) \wedge (q \rightarrow r))$ .
2. Express the negation of the sentence  $\forall x, y \exists z (P(x, y) \rightarrow Q(z)) \vee (P(x, y) \rightarrow R(z))$ . Do not use  $\rightarrow$  in your final answer.
3. Define  $P(x, y)$  to be the predicate function "Person  $x$  is *as tall as, or taller* than person  $y$ ". Determine whether each of the following quantified sentences is true or false. You may assume the domain of discourse is the set of people in this classroom.

(a)  $\exists x, y P(x, y)$

(b)  $\exists x \forall y P(x, y)$

(c)  $\exists y \forall x P(x, y)$

(d)  $\forall x, y P(x, y)$

(e)  $\forall x \exists y P(x, y)$

(f)  $\forall y \exists x P(x, y)$

4. State the converse, inverse, and contrapositive to the sentence: If yesterday was Sunday or tomorrow is Tuesday, then today is Monday. You are not allowed to accomplish this by placing: 'it is not true' in front of either the hypothesis, the conclusion, or the entire sentence.
5. For which universe of discourses is the following quantified sentence true:  $\forall y \exists x (y = x^2)$ . Your choices are  $\mathbb{R}$ ,  $\mathbb{R}^+$ ,  $\mathbb{N}$ , and  $\mathbb{Q}^+$ . You may assume both  $x$  and  $y$  belong to the same universe of discourse.
6. State, in argument form, the following valid arguments. (a) Modus Ponens, (b) Modus Tollens, and (c) Hypothetical Syllogism.
7. Determine, by any method you so choose, if the following argument is valid.

$$\begin{array}{l} p \\ (p \vee q) \rightarrow r \\ \hline q \rightarrow \sim r \\ \hline \therefore \sim q \end{array}$$

8. Construct a sentence  $P$  of three variables  $p$ ,  $q$ , and  $r$  whose truth table's final column is as given below:

$p$	$q$	$r$	$P$ 's final column
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T