

Math 3283 - Foundations of Mathematics

Exam 2 - 2019.04.05

Name: _____

1. What should you suppose in order to prove the proposition:

For any positive integers n and m , if at least one of n or m is even, then nm is not prime.

using each of the following methods of proof:

- (a) direct proof.
- (b) proof by contradiction.
- (c) proof by contraposition.

2. Disprove the following conjecture:

$$\forall \varepsilon > 0 \exists \delta > 0 \forall x \left(0 < |x - 1| < \delta \rightarrow |x^2 + 1| < \varepsilon \right)$$

3. Prove the following conjecture:

If n^2 is even, then n is even.

4. Recall that $\sum_{k=1}^n k = \frac{n(n+1)}{2}$. Furthermore, the sum of the first n odd integers can be expressed by the sum:

$\sum_{k=1}^n 2k - 1$. Prove by induction the following conjecture:

For any positive integer n , the sum of the first n odd integers is the perfect square n^2 .

5. Prove the following conjecture by the method of cases:

$$\forall x \in \mathbb{R} \ x + |x - 3| \geq 3.$$