Math 2283 - Honors Logic

Homework - Chapter 2

Name: _____

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1. Consider the following two sentential functions: $x < 3$, and $x > 3$. (a) Form the conjunction of the negations of the sentential functions, and (b)determine which numbers satisfy the compound sentence.

2. In which of its two meanings does the word "*or*" occur in the following sentences:
   (a) *Two ways were open to him: to betray his country or to die.*
   (b) *If I earn a lot of money or win the sweepstake, I shall drop logic class.*
   (c) *You can have rhubarb pie or chocolate mousse cake for dessert.*
   (d) *I can go to the concert tonight or study for logic class..*

3. Consider the following conditional sentences:
   (a) *If wishes were horses, beggars could ride.*
   (b) *If a number x is divisible by* 2 *and by* 6*, then it is divisible by* 12.
   (c) *If* 18 *is divisible by* 3 *and* 4*, then* 18 *is divisible by* 6.

Which of the above implications are true and which are false from the point of view of mathematical logic? In which cases does the question of meaningfulness and of truth or falsity raise any doubt from the standpoint of ordinary logic?

4. Put the following theorem into the form of an ordinary conditional sentence:

   For a triangle to be equilateral, it is sufficient that the angles of the triangle be congruent.

5. Determine if the condition:
$$x \cdot y > 4$$
is necessary or sufficient for the validity of
$$x > 2 \text{ and } y > 2.$$

6. Give alternative formulations for the following sentence:

   A number x is divisible by 10*, if and only if, x is divisible both by* 2 *and by* 5.

7. Determine which of the following sentences are true:
   (a) *A triangle is isosceles if, and only if, all the altitudes of the triangle are congruent.*
   (b) *The fact that $x \neq 0$ is necessary and sufficient for x^2 to be a positive number.*

8. Formulate the definition of the term "*parallel*"; what terms (from the domain of geometry) have to presupposed for this purpose? (Do not restrict to your definition to lines in the plane.)

9. Translate the following symbolic expressions into ordinary language:
   (a) $[(\sim p) \to p] \to p$
   (b) $[(\sim p) \vee q] \leftrightarrow (p \to q)$
   (c) $[\sim (p \vee q)] \leftrightarrow (p \to q)$
   (d) $(\sim p) \vee [q \leftrightarrow (p \to q)]$

   Direct special attention to the difficulty in distinguishing in ordinary language the three last expressions.

10. Construct truth tables for all sentential functions given in Exercise 9 and state which sentences are tautologies and which are not.

11. Formulate the following expressions in logical symbolism:
(a) *If r follows from p and if r follows from q, then r follows from p or q.*
(b) *If p implies q, then: q implies r, then p implies r.*

12. Verify by the method of truth tables that the following sentences are tautologies:
(a) $(p \leftrightarrow q) \rightarrow \{[(r \wedge p) \rightarrow s] \rightarrow [(r \wedge q) \rightarrow s]\}$
(b) $(p \rightarrow q) \rightarrow [(q \rightarrow p) \rightarrow (p \leftrightarrow q)]$
(c) $[\,(p \rightarrow r\,) \wedge (\,q \rightarrow r\,)\,] \leftrightarrow [\,(\,p \vee q\,) \rightarrow r]$

13. For the following sentence, state the three corresponding conjugate sentences:

If a quadrangle is a rectangle, then a circle can be circumscribed about it.

14. For the sentence in Exercise 13, determine which of the conjugate sentences are true.

15. Give an example of four conjugate sentences which are all false. In other words, for $p \rightarrow q$, find a single example of sentences p and q such that all the conjugate sentences are false.