Math 2283 - Honors Logic

Midterm - 2019.10.15

Name: ______________________________

Relation Properties:
A relation R is *co-reflexive* if and only if $\forall\, x, y,\ xRy \rightarrow x = y$.

A relation R is *left unique* if and only if $\forall\, x_1, x_2, y,\ x_1Ry \wedge x_2Ry \rightarrow x_1 = x_2$.

A relation R is *right quasi-reflexive* if and only if $\forall\, x, y,\ xRy \rightarrow yRy$.

1. If you look at problem 3 below, you are to prove a conditional sentence. In particular, note that in Theorem 1, you are to prove R is co-reflexive assuming R is right quasi-reflexive and left unique. The definition of co-reflexive is itself the conditional sentence $\forall\, x, y,\, xRy \rightarrow x = y$. One might be able to derive the entire definition of co-reflexive using only the assumptions of right quasi-reflexiveness and left uniqueness. However, it is easier if one assumes the hypothesis of the property of co-reflexiveness, and then simply derives $x = y$ using the properties of right quasi-reflexiveness and left uniqueness. To do this, one must argue that this is indeed a valid approach. We do this in parts (a) and (b) as follows:

(a) [10 pts] Construct a truth table for the following sentential function: $[(p \wedge q \wedge r) \rightarrow s] \longleftrightarrow [(p \wedge q) \rightarrow (r \rightarrow s)]$

(b) [10 pts] Using the result from (a), argue why the aforementioned approach to proving certain conditional theorems is valid (and can thus be used in problems 3 below).

2. [5 pts $\times$ 2] Express the properties of co-reflexiveness and left uniqueness, as defined above, using ONLY the calculus of relations. For reference, see problem 9 from the Chapter 5 HW in the textbook.

3. [12 pts $\times$ 2] Prove the following theorems of relations:

Theorem 1: If a relation R is right quasi-reflexive and left unique on a class K, then the relation R is co-reflexive on the class K.

Theorem 2: If a relation R is co-reflexive on a class K, then the relation R is symmetric on the class K.

4. [5 pts $\times$ 2] Express, in terms of elements, the definition of the empty and universal sets, $\emptyset$ and U, respectively by filling in the blanks of the following two definitions.

$K = \emptyset \overset{def}{\longleftrightarrow} \forall\, x$ ______________

$K = U \overset{def}{\longleftrightarrow} \forall\, x$ ______________

5. [12 pts $\times$ 3] Define the class operation '$-$' on two classes K and L as follows:

$$x \in K - L \overset{def}{\longleftrightarrow} x \in K \wedge \sim x \in L$$

Prove the following class based theorem:

Theorem 1: $(K - L) \cup M = (K \cup M) - (L - M)$

Theorem 2: $K - (L \cup M) = (K - L) \cap (K - M)$

Theorem 3: $K \subseteq L \longrightarrow K - L = \emptyset$