Chapter 4: Integration

4.1 Antidifferentiation

$F(x)$ is an antiderivative of $f(x)$

i) $F'(x) = f(x)$

note: $\frac{d}{dx} x^5 = 5x^4$

$\frac{d}{dx} x^5 + 4 = 5x^4$

$\frac{d}{dx} x^5 - 2 = 5x^4$

If $F'(x) = f(x)$,

then $F(x) + C$ is also

an antiderivative of $f(x)$

$\frac{d}{dx} e^{3x} = 3e^{3x}$

so if $F(x) = e^{3x}$, $F(x)$ is an

antiderivative for $f(x) = 3e^{3x}$.

Since $F'(x) = f(x)$

But $F(x) + C = e^{3x} + C$ is also

$F(x) = e^{3x}$

$f(x) = 3e^{3x}$

$F'(x) = f(x) \longrightarrow$ $F(x)$ is an anti-derivative of $f(x)$.

$F(x) = e^{3x} + 12$, $F'(x) = f(x)$, so $F(x)$ is still an antiderivative of $f(x)$

$- 4$

$+ 7$

$\therefore + C$

Notation: $\int 3e^{3x}\,dx = e^{3x} + C_1$

$$\int \underbrace{5x^4}_{f(x)}\,dx = \underbrace{x^5}_{F(x)} + C$$

note: $F'(x) = f(x)$

Notation: $\int f(x)\,dx = F(x) + C$, where $F'(x) = f(x)$

$$\int \frac{1}{x^2} + 3x \, dx = -\frac{1}{x} + \frac{3}{2}x^2 + C_1$$

Check: $\frac{d}{dx}\left(-\frac{1}{x} + \frac{3}{2}x^2 + C_1\right)$

$$= \frac{d}{dx}\left(-x^{-1} + \frac{3}{2}x^2 + C_1\right)$$

$$= -(-1)x^{-2} + 3x + 0$$

$$= \frac{1}{x^2} + 3x$$

$$\int 0 \, dx = C$$

$$\int k \, dx = k \cdot x + C$$

$$\int f(x) + g(x) \, dx = \int f(x) \, dx + \int g(x) \, dx$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$

ex: $\int x^2 \, dx = \frac{1}{3} x^3 + C$

$\int x^{-5} \, dx = \frac{1}{-4} x^{-4} + C$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad \text{for } n \neq -1$$

$$\int \frac{1}{x} dx = \ln(|x|) + C$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} = \ln(x) + C \quad \text{if } x > 0$$

$$= \ln(-x) + C \quad \text{if } x < 0$$

$$\int 3x^2 - \frac{4}{x} + \frac{6}{x^5} + 1 \, dx = x^3 - 4\ln(x) - \frac{6}{4}x^{-4} + x + C$$

$$\frac{d}{dx}\left(x^3 - 4\ln(x) - \frac{6}{4}x^{-4} + x + C\right)$$

$$= 3x^2 - 4 \cdot \frac{1}{x} - \frac{6}{4}(-4)x^{-5} + 1 + 0$$