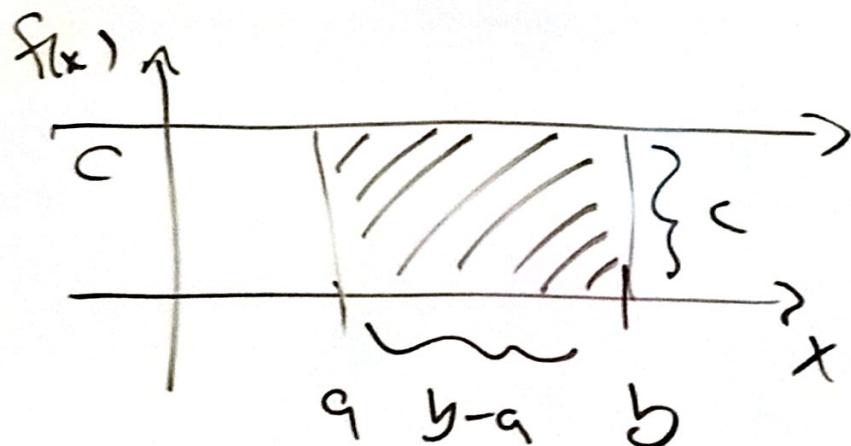
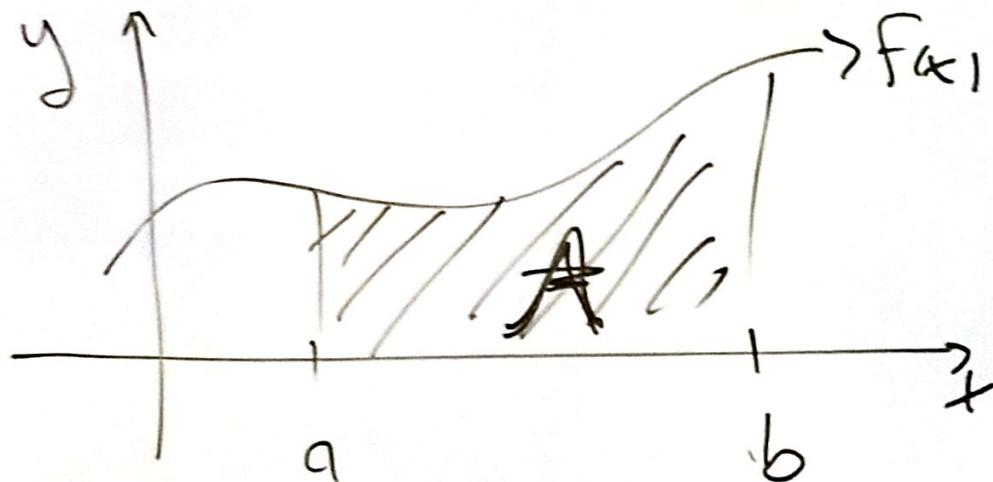
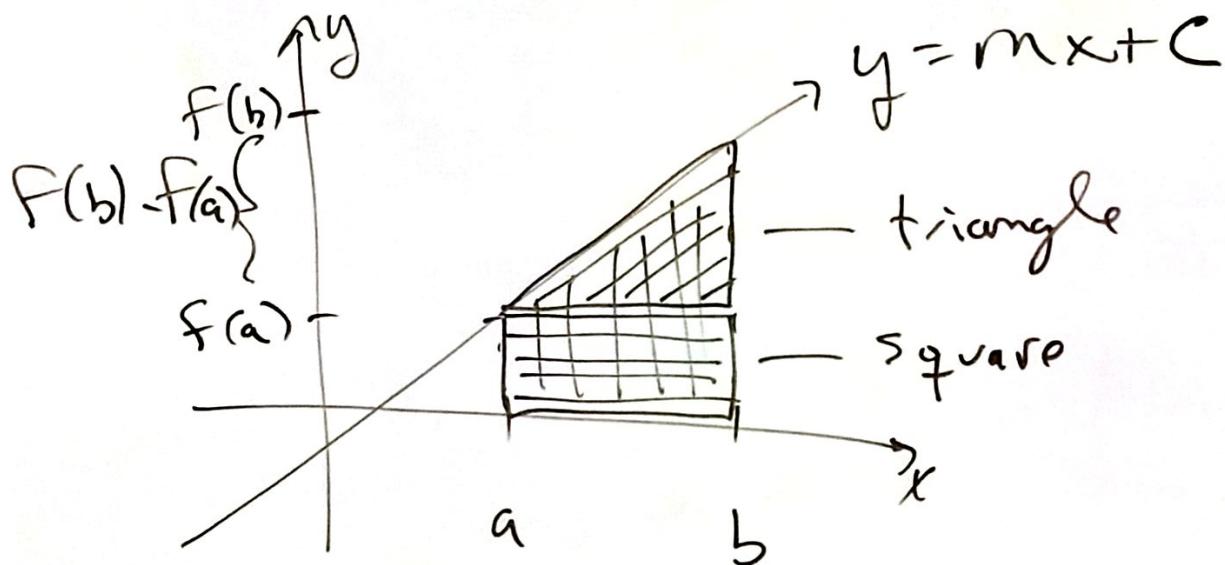


## 4.2 Areas under Curves.



for constant function  $f(x)=c$ ,  $A=(b-a)c$



base for both  $\square$  and  $\triangle$  is  $b-a$

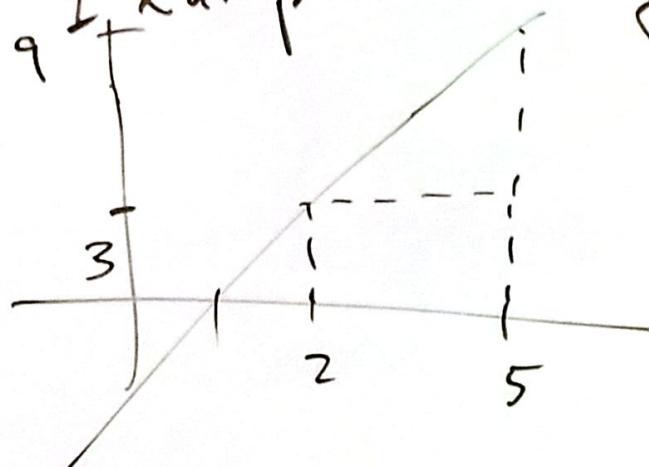
height for  $\square$  is  $f(a)=ma+c$

height for  $\triangle$  is  $f(b)-f(a)=m(b-a)$

total area:  $(b-a) \cdot (ma+c + \frac{1}{2}m(b-a))$

base  $\square$  height of  $\square$   $+ \frac{1}{2}$  height of  $\triangle$

Example:  $y = 2x - 1$  on  $[2, 5]$



$$\begin{aligned} \text{Area: } & 3 \cdot 3 + \frac{1}{2} \cdot 3 \cdot 6 \\ & = 9 + 9 = 18 \end{aligned}$$

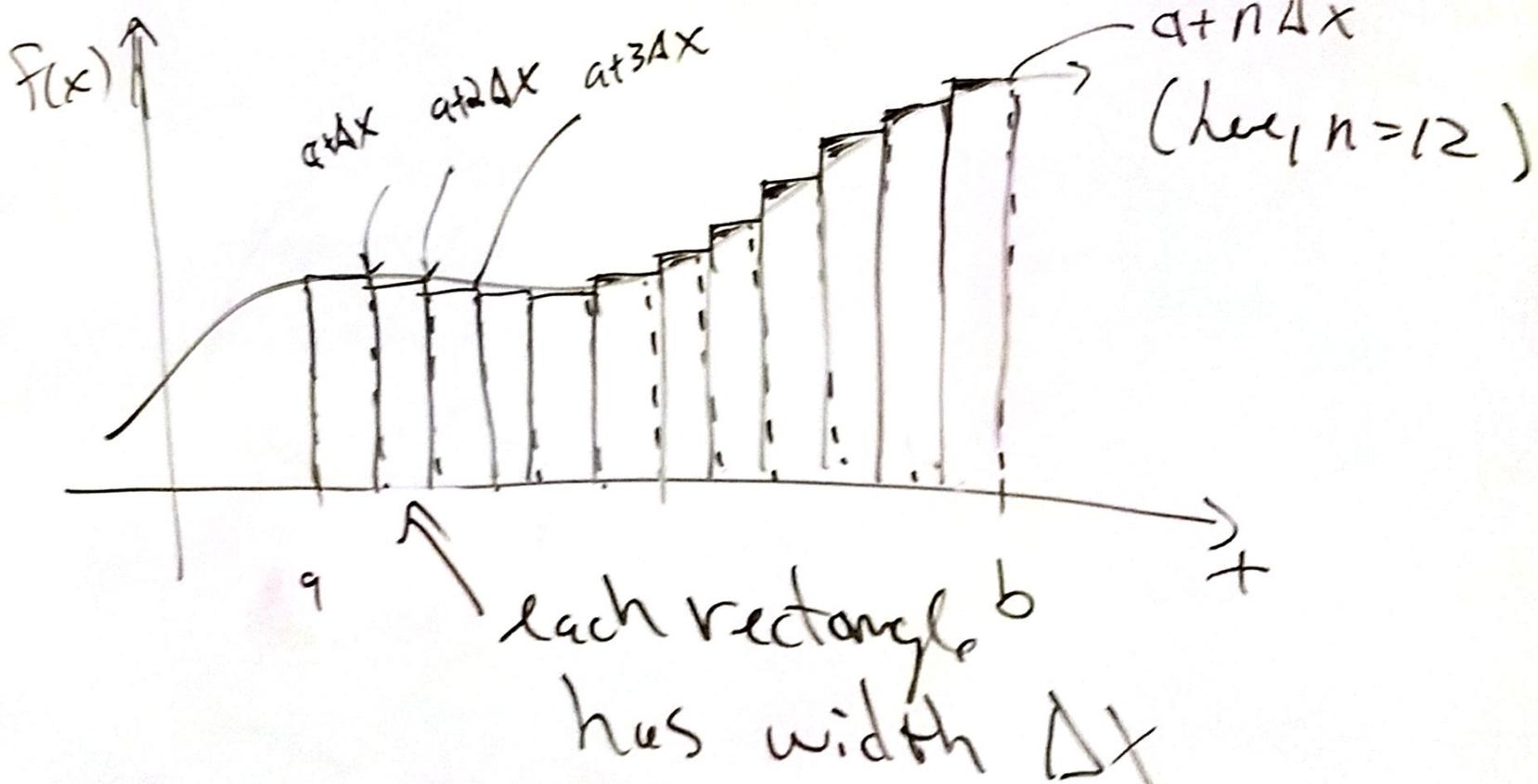
Summation Notation:

$$\sum_{k=0}^n f_k = f_0 + f_1 + f_2 + \dots + f_n$$

Greek capital sigma

Ex:

$$\sum_{k=1}^{10} k = 1 + 2 + 3 + \dots + 10$$
$$= \frac{10 \cdot (10+1)}{2} = 55$$
$$\sum_{k=1}^{10} k^2 = 1^2 + 2^2 + 3^2 + \dots + 10^2$$
$$= \frac{10 \cdot (10+1) \cdot (2 \cdot 10 + 1)}{6}$$
$$\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n-1 + n$$
$$= \frac{n \cdot (n+1)}{2}$$
$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$



Area under  $f(x)$  on  $[a, b]$  is

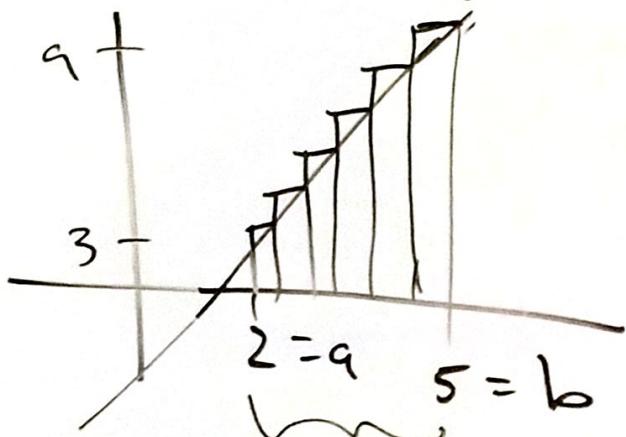
~~height · base~~

$$\text{Area} \approx \underline{\sum_{k=1}^n f(atk\Delta x) \cdot \Delta x} + f(at2\Delta x) \cdot \Delta x + \dots +$$

$$\approx \sum_{k=1}^n f(atk\Delta x) \cdot \Delta x$$

~~height · base~~

Back to  $y = 2x - 1$  on  $[2, 5]$



Area:  $\sum_{k=1}^n f(a+k\Delta x) \cdot \Delta x$

$$n=10, \Delta x = \frac{3}{10} = \frac{b-a}{n}$$

10 rectangles of  
same width

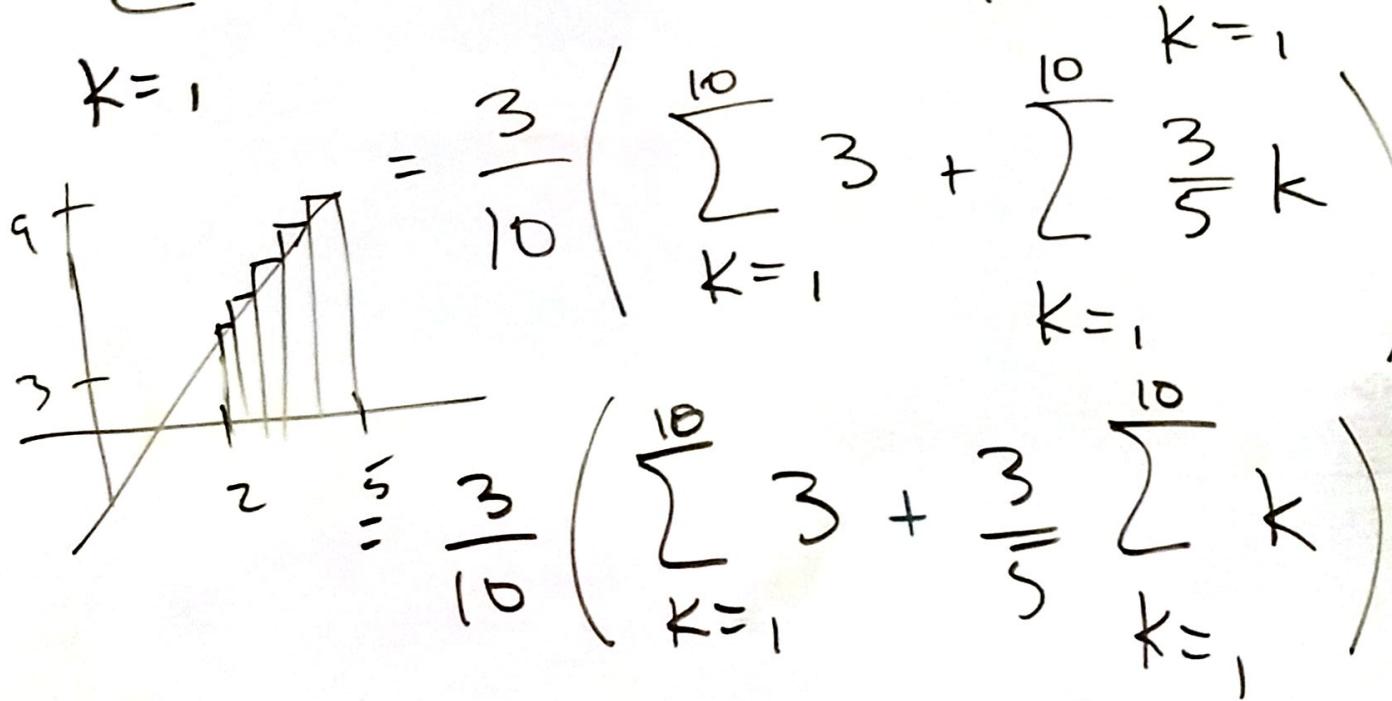
$$\text{Area} = \sum_{k=1}^{10} f(2+k \cdot \frac{3}{10}) \cdot \frac{3}{10}$$

but  $f(x) = 2x - 1$ , so

$$\begin{aligned} f(2 + \frac{3k}{10}) &= 2(2 + \frac{3k}{10}) - 1 \\ &= 4 + \frac{3}{5}k - 1 \\ &= 3 + \frac{3}{5}k \end{aligned}$$
$$\sum_{k=1}^{10} \left(3 + \frac{3}{5}k\right) \cdot \frac{3}{10}$$

Back to  $y = 2x - 1$  on  $[2, 5]$

$$\sum_{k=1}^{10} \left(3 + \frac{3}{5}k\right) \cdot \frac{3}{10} = \frac{3}{10} \cdot \sum_{k=1}^{10} 3 + \frac{3}{5}k$$



$$= \frac{3}{10} \left( 30 + \frac{3}{5} \sum_{k=1}^{10} k \right)$$

$$= 9 + \frac{3 \cdot 33}{10} = 9 + \frac{99}{10} = 18.9$$

Back to  $y = 2x - 1$  on  $[2, 5]$

using  $n$  rectangles

$$\text{Area} \approx \sum_{k=1}^n f(2 + k\Delta x) \cdot \Delta x$$

$$\text{here, } \Delta x = \frac{5-2}{n} = \frac{3}{n}, f\left(2 + \frac{3k}{n}\right) =$$

$$\begin{aligned}\text{Area: } & \sum_{k=1}^n \left(3 + \frac{6k}{n}\right) \cdot \frac{3}{n} & 2 \cdot \left(2 + \frac{3k}{n}\right) - 1 \\ & = 4 + \frac{6k}{n} - 1 \\ & = 3 + \frac{6k}{n}\end{aligned}$$