

# Irreflexive & Transitive

$\rightarrow$  Asymmetrical  
( $\forall x \sim xRx \wedge \forall x, y, z \ xRy \wedge yRz \rightarrow xRz$ )

$\rightarrow \forall x, y \ xRy \rightarrow \sim yRx$

Different Approach:

assume hypothesis of consequent:  $xRy$

derive  $\sim yRx$

using Irreflexive & Transitive

as axioms.

$$x \leq y \stackrel{\text{def}}{\iff} x < y \vee x = y$$

Thm D:  $(x < y \wedge y \leq z) \rightarrow x \leq z$

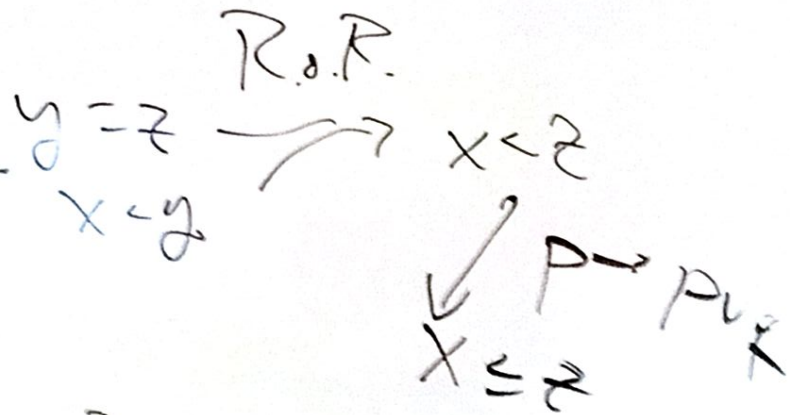
Ax 4:  $x < y \wedge y < z \rightarrow x < z$

①  $x < y \wedge y \leq z$

$$\left[ \begin{array}{l} x < y \\ y \leq z \end{array} \right.$$

$$\left[ \begin{array}{l} y < z \vee y = z \\ z = y \vee z > y \end{array} \right. \quad p \rightarrow p \vee q$$

$$x < y \wedge y < z \rightarrow x < z \quad \rightarrow \quad z = x \vee x < z \rightarrow x \leq z$$



Thm F:  $(x \leq y \wedge y < z \wedge z \leq t) \rightarrow x < t$

$x \leq y \wedge y < z \wedge z \leq t$

$x \leq y \wedge y < z$

THE:  $x < z$  P.O.D.

$q: z \leq t \downarrow ?$

$x < z \wedge z \leq t$

$\downarrow$  Thm D  $y: z, z, t$   
P.O.D.

$x < t$



$=, <, >, \neq, \leq, \geq$

$$x < y \rightarrow x \neq y$$

by S.L.O.T.

$< \subset \neq$

$$S \subset R \stackrel{\text{def}}{\iff} x S y \rightarrow x R y$$

$= \subset <$

$$x < y \rightarrow x \neq y$$

$$x < y \rightarrow x \leq y$$

$> \subset \geq$

by def of  $\leq$