Irreflexive & transitive

$\longrightarrow$ Asymmetrical

$$(\forall x\ \neg xRx \wedge \forall x,y,z\ xRy \wedge yRz \rightarrow xRz)$$

$$\longrightarrow \forall x,y\ xRy \rightarrow \neg yRx$$

Different Approach:

assume hypothesis & consequent: xRy

derive $\neg yRx$ using Irreflexive & transitive as axioms.

$$x \le y \xleftrightarrow{\text{def}} x < y \lor x = y$$

Thm D: $(x < y \land y \le z) \to x \le z$

Ax 4: $x < y \land y < z \to x < z$

① $x < y \land y \le z$

R.o.R.

$y = z \longrightarrow x < z$

$x < y$

$x < y$

$y \le z$

$P \to P \lor q$

$y < z \lor y = z \quad p \to p \lor q$

?

$x \le z$

$x < y \land y < z \to x < z \longrightarrow x \le z$

Thm F_p: $(x \le y \wedge y < z \wedge z \le t) \to x < t$

$$\overbrace{x \le y \wedge y < z}^{p} \wedge \overbrace{z \le t}^{q}$$

$$\overbrace{x \le y \wedge y < z}^{\sigma p}$$

ThE: $x < z$ P.O.D.

q: $z \le t$ $\big\downarrow$?

$$x < z \wedge z \le t$$

$\big\downarrow$ Thm D   P.O.D. $y : z,\ z : t$

$$x < t$$

$=, <, >, \neq, \leq, \geq$

$x < y \rightarrow x \neq y$ by S.L.O.T.

$< \subset \neq$

$S \subset R \overset{def}{\longleftrightarrow} xSy \rightarrow xRy$

$= \not\subset <$

$x < y \rightarrow x \neq y$

$x < y \rightarrow x \leq y$ by def of $\leq$

$< \subset \leq$