Math 2283 - Honors Logic
Homework - Chapter 1

Name: _____

1. A statement which can be determined to be true or false is called what?

2. If you substitute constants for variables in a designatory function, what do you end up with?

3. If you substitute constants for variables in a sentential function, what do you end up with?

4. What do the symbols $\forall$ and $\exists$ represent?

5. Determine which of the following expressions are sentential functions, and which are designatory functions:
   (a) the sum of the numbers x and 2
   (b) the integer after the number w
   (c) the mother of x and y
   (d) x is the mother of y
   (e) t is a day of the week after Wednesday

6. Determine whether each of the following are constants, sentential functions, designatory functions, or sentences.
   (a) The capital city of Oklahoma.
   (b) City x is the capital of Oklahoma.
   (c) City x.
   (d) The magnolia capital of Oklahoma.
   (e) The magnolia capital of Oklahoma is Durant.
   (f) A state which shares a border with Oklahoma.
   (g) A state which shares a border with Oklahoma is Texas.
   (h) Student x is enrolled in Honor's Logic.
   (i) The students enrolled in the Honor's Logic course.
   (j) Professor x.
   (k) Professor x is the instructor of record for Honor's Logic.

7. Replace the sentence: '*Some snakes are venomous.*' by one which has the same meaning but is formulated with the help of quantifiers and variables.

8. By writing quantifiers containing the variables "x" and "y" in front of the sentential function: x is the father of y, it is possible to obtain various sentences from it, for instance

$$\forall\, x, y \ (\ x \text{ is the father of } y\ )$$
$$\forall\, x \ \exists\, y \ (\ x \text{ is the father of } y\ )$$
$$\exists\, y \ \forall\, x \ (\ x \text{ is the father of } y\ )$$

Formulate the remaining three quantified sentences, and determine which of them are true. Here you should assume "x" and "y" stand for names of human beings.

9. Determine which of the following quantified sentences are true, and which are false. Here, you may assume that x belongs to the set of U.S. cities, and y belongs to the set of states of the U.S.
   (a) $\exists\, x, y$ such that x is capital city of the state y.
   (b) $\exists\, x \ \forall\, y$ such that x is capital city of the state y.
   (c) $\forall\, y \ \exists\, x$ such that x is capital city of the state y.
   (d) $\exists\, y \ \forall\, x$ such that x is capital city of the state y.
   (e) $\forall\, x \ \exists\, y$ such that x is capital city of the state y.

(f) $\forall\, x, y$ x is capital city of the state y.

10. Determine which constants satisfy the following sentential functions:
   (a) $\exists\, x,$ x is the father of y
   (b) $\exists\, y,$ x is the sister of y
   (c) $\exists\, y$ such that y is the brother of x.

11. If we consider only people who are enrolled in this course, determine which of the following quantified statements are true, and which are false.
   (a) $\forall x\, \exists y,$ x is taller than y
   (b) $\exists x\, \forall y,$ x is taller than y
   (c) $\exists x, y,$ x is taller than y
   (d) $\exists y\, \forall x,$ x is taller than y