Math 2283 - Honors Logic

Homework - Chapter 2

Name: _____

---

1. Consider the following two sentential functions: x is odd, and x has a factor other than 1 and itself. (a) Form the conjunction of the negations of the sentential functions, and (b)determine which numbers satisfy the compound sentence.

2. In which of its two meanings does the word "*or*" occur in the following sentences:
   (a) *Two ways were open to him: to betray his country or to die.*
   (b) *To be or not to be, that is the question.*
   (c) *You can have rhubarb pie or chocolate mousse cake for dessert.*
   (d) *I can go to the concert tonight or study for logic class.*

3. Consider the following conditional sentences:
   (a) *If wishes were horses, beggars could ride.*
   (b) *If a number x is a prime number, then x is odd.*
   (c) *If* 18 *is divisible by* 3 *and* 4, *then* 18 *is divisible by* 6.
   (d) *If turnips were watches, then I'd wear one by my side.*

Which of the above implications are true and which are false from the point of view of logic? In which cases does the question of meaningfulness and of truth or falsity raise any doubt from the standpoint of ordinary logic?

4. Put the following theorem (Pascal's Theorem) into the form of an ordinary conditional sentence:

For three pairs of opposite sides of the hexagon (extended if necessary) meet at three points which lie on a straight line it is sufficient that six arbitrary points are chosen on a conic (which may be an ellipse, parabola or hyperbola in an appropriate affine plane) and joined by line segments in any order to form a hexagon.

5. Determine if the condition:

$$x^2 + y^2 < 1$$

is necessary or sufficient for the validity of

$$-1 < x < 1 \text{ and } -1 < y < 1.$$

6. Determine which of the following sentences are true:
   (a) *A triangle is acute if, and only if, the three altitudes of the triangle intersect in a single point inside the triangle.*
   (b) *The fact that x is prime and greater than 2 is necessary and sufficient for x to be odd.*

If a sentence is not true, determine to rewrite the sentence so that it is true by writing the sentence as a conditional sentence.

7. Construct truth table for the following sentences:
   (a) $[(\sim p) \to p] \to p$
   (b) $[(\sim p) \vee q] \leftrightarrow (p \wedge \sim q)$
   (c) $[\sim (p \wedge q)] \vee (p \to q)$
   (d) $(p \leftrightarrow q) \to \{[(r \wedge p) \to s] \to [(r \wedge q) \to s]\}$
   (e) $(p \to q) \to [(q \to p) \to (p \leftrightarrow q)]$
   (f) $[\,(p \to r\,) \wedge (\,q \to r\,)\,] \leftrightarrow [\,(\,p \vee q\,) \to r]$

8. Formulate the definition of the term "*parallel*"; what terms (from the domain of geometry) have to presupposed for this purpose? (Do not restrict to your definition to lines in the plane.)

9. Formulate the following expressions in logical symbolism:
   (a) *If r follows from p and if r follows from q, then r follows from p or q.*
   (b) *If p implies q, then: q implies r, then p implies r.*

10. For the following sentence, state the three corresponding conjugate sentences:

   If a point P is on the perpendicular bisector of a segment, then P is equidistant from the endpoints of the segment.

11. For the sentence in Exercise 10, determine which of the conjugate sentences are true.

12. Give an example of four conjugate sentences which are all false. In other words, for $p \to q$, find a single example of sentences p and q such that all the conjugate sentences are false.