Math 2283 - Honors Logic

Homework - Chapter 4

Name: _____

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1. Consider the following sets:

$A = \{\, x \mid x \text{ is a US state bordering the Atlantic Ocean.} \,\}$

$B = \{\, x \mid x \text{ is a US state bordering the Pacific Ocean.} \,\}$

$C = \{\, x \mid x \text{ is a US state which has a coast on the Gulf of Mexico.} \,\}$

$D = \{\, x \mid x \text{ is a US state which shares a border with Canada.} \,\}$

$E = \{\, \text{Texas, Oklahoma, Arkansas} \,\}$

$F = \{\, x \mid x \text{ is a US state which shares no border with any other US state.} \,\}$

$G = \{\, x \mid x \text{ is a US state whose has as part of its border the Red River of the South.} \,\}$

Relate the following pairs of sets using the relations: subset, superset, overlap, disjoint, and equal.

(a) A B (b) A C (c) F B

(d) E G (e) D B (f) C G

2. Determine which of the fundamental relations (cf. Theorem 4.1) hold between the following pairs of intervals:

(a) $[2, 4], [5, 8]$ (b) $[4, 7], [3, 5]$ (c) $[4, 7], [4, 7]$

(d) $[1, 3], [-2, 4]$ (e) $[1, 3], [1, 2]$ (f) $[0, 1], [-2, 1]$

(g) $[1, 7], [-2, 4]$ (h) $[-2, 10], [1, 2]$ (i) $[1, 1], [-1, 2]$

3. Draw two boxes K and L so that they stand in one of the following relations (cf. Theorem 4.1):

(a) $K = L$

(b) the box K is a proper subclass of the box L

(c) the box L is a proper subclass of the box K

(d) the boxes K and L overlap

(e) the boxes K and L are disjoint

4. Which of the cases from problem 3 are eliminated if K and L are congruent?

5. Which of the cases from problem 3 are eliminated if we consider only the perimeters of K and L (hence K and L are rectangles)?

6. Is the following sentence (which has the same structure as the Law 4.4, the Law of Class Transitivity for Inclusion, of Section 4.4) true?

If K is disjoint from L and L is disjoint from M, then K is disjoint from M.

7. Convert the following logical sentence into set notation so that there are no logical connectives or element symbols, instead only set operations and set relations.

$$\sim (x \in K \vee x \in L) \longleftrightarrow (\sim x \in K \wedge \sim x \in L)$$

8. Let $\triangle ABC$ be an arbitrary triangle, with an arbitrary point D lying on line segment $\overline{BC}$. Express your answers to the following two questions in formulas:

(a) What figures are formed by the union of the two triangles $\triangle ABD$ and $\triangle ACD$?

(b) What figures are formed by the intersection of the two triangles $\triangle ABD$ and $\triangle ACD$?

9. Represent an arbitrary square:

(a) as the union of two trapezoids,
(b) as the intersection of two triangles.

10. Let K and L be two arbitrary classes. What classes are $K \cup L$ and $K \cap L$ in the case $K \subseteq L$?

11. Let K be an arbitrary class. Determine each of the following classes:
(a) $K \cup U$ (b) $K \cap U$ (c) $\emptyset \cup K$ (d) $\emptyset \cap K$

Prove each of the following laws, given arbitrary classes K, L, and M.

12. **Law of Simplification for Union**: $K \subseteq K \cup L$
13. **Law of Simplification for Intersection**: $K \cap L \subseteq K$
14. **Distributive Law of Intersection over Union**: $K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$
15. **Distributive Law of Union over Intersection**: $K \cup (L \cap M) = (K \cup L) \cap (K \cup M)$
16. **The Law of Double Complement**: $(K')' = K$
17. **De Morgan's Law for Union**: $(K \cup L)' = K' \cap L'$
18. **De Morgan's Law for Intersection**: $(K \cap L)' = K' \cup L'$

19. Consider the following three sets:
(a) The set of all natural numbers greater than 0 and less than 4.
(b) The set of all rational numbers greater than 0 and less than 4.
(c) The set of all irrational numbers greater than 0 and less than 4.

Which of these set are finite and which are infinite?

20. Define $U = \mathbb{N}$ to be the set of all positive integers, and $\mathbb{K} = \{6, 12, 18, \ldots\}$. Determine *all* values which make the following sentential function true.

$$\forall\, x \in \mathbb{K}\ \ (x/y \in \mathbb{N} \wedge y \in \mathbb{K}')$$