Math 2283 - Honors Logic

Homework - Chapter 5

Name: ______________________________

A relation R is *co-reflexive* if and only if $\forall\, x, y,\ xRy \to x = y$.

A relation R is *left unique* if and only if $\forall\, x_1, x_2, y,\ x_1Ry \wedge x_2Ry \to x_1 = x_2$.

A relation R is *right quasi-reflexive* if and only if $\forall\, x, y,\ xRy \to yRy$.

1. If you look at problem 3 below, you are to prove a conditional sentence. In particular, note that in Theorem 2, you are to prove R is co-reflexive assuming R is right quasi-reflexive and left unique. The definition of co-reflexive is itself the conditional sentence $\forall\, x, y,\ xRy \to x = y$. One might be able to derive the entire definition of co-reflexive using only the assumptions of right quasi-reflexiveness and left uniqueness. However, it is easier if one assumes the hypothesis of the property of co-reflexiveness, and then simply derives $x = y$ using the properties of right quasi-reflexiveness and left uniqueness. To do this, one must argue that this is indeed a valid approach. We do this in parts (a) and (b) as follows:

(a) Construct a truth table for the following sentential function: $[(p \wedge q \wedge r) \to s] \longleftrightarrow [(p \wedge q) \to (r \to s)]$

(b) Using the result from (a), argue why the aforementioned approach to proving certain conditional theorems is valid (and can thus be used in problems 3 below).

2. Express the properties of co-reflexiveness and left uniqueness, as defined above, using ONLY the calculus of relations. For reference, see problem 9 from the Chapter 5 HW in the textbook.

3. Prove the following theorems of relations:

Theorem 1: If a R is asymmetrical on a class K, then the relation R' is reflexive and connected on the class K
.

Theorem 2: If a relation R is right quasi-reflexive and left unique on a class K, then the relation R is co-reflexive on the class K.

Theorem 3: If a relation R is co-reflexive on a class K, then the relation R is symmetric on the class K.

4. Prove each of the following relation among relations is satisfied by arbitrary relations R, S, and T.

(a) $\breve{(R/S)} = \breve{S}/\breve{R}$
(b) $R/(S/T) = (R/S)/T$

5. Give an example of a relation which satisfies each of the following:
(a) R is neither reflexive nor irreflexive.
(b) S is neither symmetric nor asymmetric.
(c) T is neither transitive nor intransitive.

6. Let $\mathbb{R}$ be the set of all real numbers, and define the following relations:

$$xRy \longleftrightarrow x \cdot y \geq 0, \quad xSy \longleftrightarrow x - y \geq 0, \qquad xTy \longleftrightarrow x - y > 0$$

(a) Which of the relations R, S, and T are reflexive?
(b) Which of the relations R, S, and T are transitive?
(c) Which of the relations R, S, and T are symmetric?
(d) Which of the relations R, S, and T are asymmetric?

(e) Which of the relations R, S, and T are connected?

7. Attempt to define the relation of '*preceding*' with respect to two words in the English language, and then show that this relation defined an ordering on the set of all words in the English language. As a hint, consider the dictionary!

8. State the property of relations expressed by the formula: $\breve{R}/R \subseteq I$

9. Which of the relations (of the form xRy) expressed by the following formulas are functions:
(a) x is the mother of y
(b) x is the daughter of y
(c) y is the mother of x
(d) x is the sibling of y
(e) y is the sibling of x

10. Show that the set of all natural numbers and the set of all odd numbers are equinumerous.