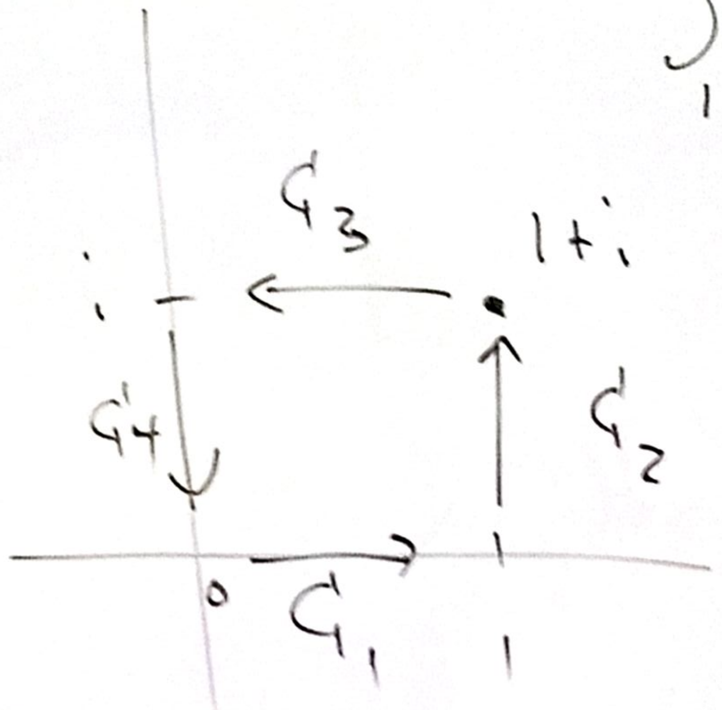
C_3 $z = x+i$, $x: 1 \to 0$

$dz = dx$

$$\int_1^0 (x-i)e^{x+i}\,dx = -\int_0^1$$



$$\int_{C_1} \bar{z}\, e^{z}\, dz$$

C_1 $z = x$ $x: 0 \to 1$

$dz = dx$ $\longrightarrow$ $\displaystyle\int_0^1 x e^x\, dx$

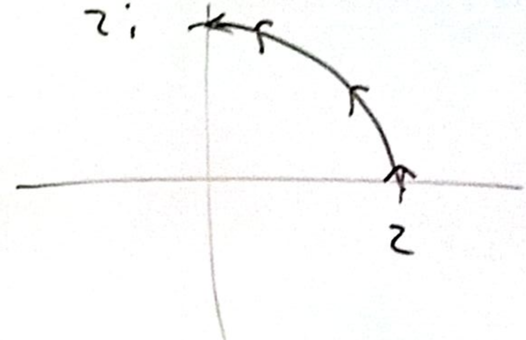
$$\int_C f(z)\,dz = \int_{t_0}^{t_1} f(z(t))\,z'(t)\,dt$$

C_2: $z = 1+iy$, $y: 0 \to 1$

$dz = i\,dy$ $\longrightarrow$ $\displaystyle\int_0^1 (1-iy)\,e^{1+iy} \cdot i\,dy$

$$\left|\int_{C_1} \frac{1}{z^2-1}\,dz\right| \le \pi\cdot\frac{1}{3}$$

C_1:



$$L = \frac{2\pi\cdot 2}{4} = \pi$$

$$M = \max_{z\in C_1} \frac{1}{|z^2-1|} = \frac{1}{\min_{z\in C_1} |z^2-1|} = \frac{1}{3}$$

on C_1, $z = 2e^{i\theta}$

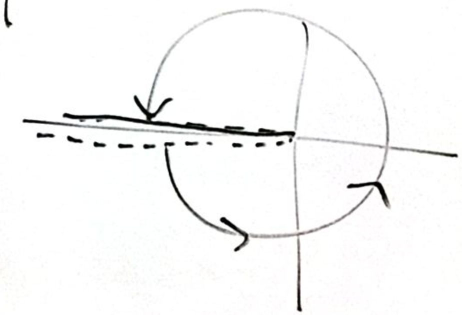
$$z^2 = 4e^{i2\theta} = 4\cos(2\theta) + 4i\sin(2\theta)$$

$$|z^2-1|^2 = (4\cos(2\theta)-1)^2 + (4\sin(2\theta))^2$$

$$= 16+1-8\cos(2\theta) \ge 9$$

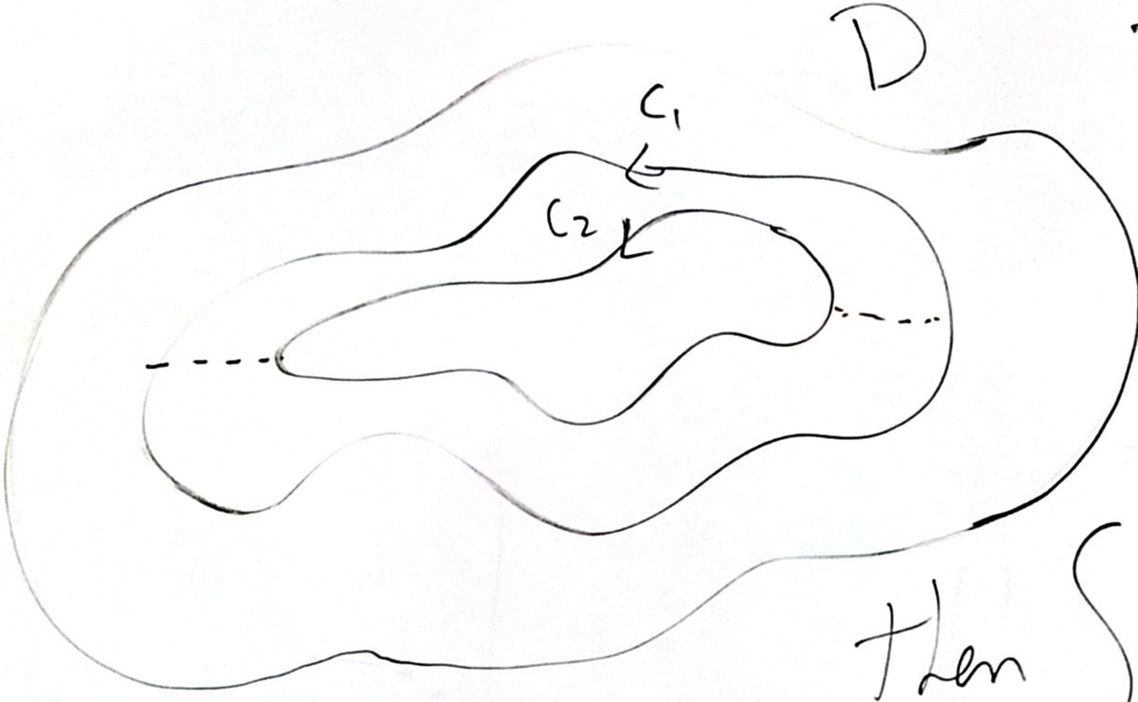
$$\mathrm{Log}(z) = \ln|r| + i\,\mathrm{Arg}(z)$$

$$-\pi < \mathrm{Arg}(z) \leq \pi$$



$$\left| \int_{C_R^+(0)} \text{stuff}\, dz \right| \leq 2\pi R \cdot M$$

$$L = 2\pi R \qquad z = Re^{i\theta} \qquad \theta: -\pi \to \pi$$



If $f(z)$ is analytic in D, and $C_1, C_2 \subset D$

then $\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz$

If $f(z)$ is analytic on D, and D is simply connected, and $C \subset D$ is simple closed,

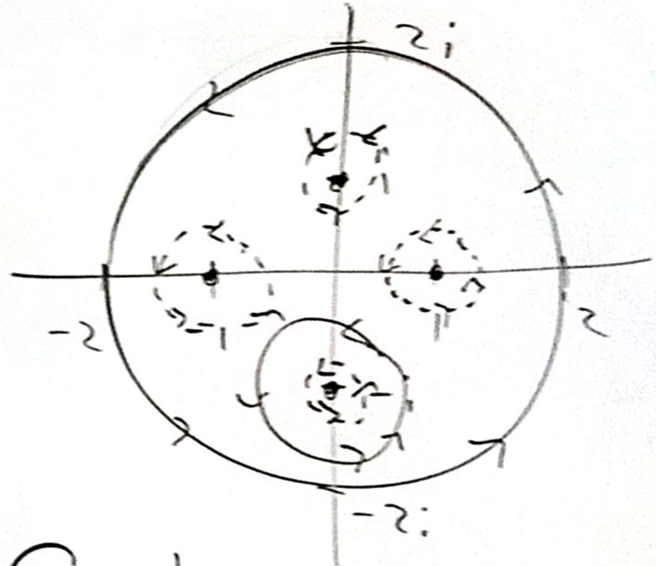
$$\int_C f(z)\,dz = 0$$

$$f(z) = \frac{1}{z-z_0}$$

$$\int_{C_1} \frac{1}{z-z_0}\,dz = 0 = \int_{C_3} \frac{1}{z-z_0}\,dz \le |f(z_1)| \cdot 2\pi\varepsilon$$

$$\int_{C_2} \frac{1}{z-z_0}\,dz = 2\pi i = \int_{C_4} \frac{1}{z-z_0}\,dz$$

$$\int_{C^+_{1/2}(0)} \frac{1}{z^4-1}\,dz$$



$$z^4-1=(z-1)(z+1)(z-i)(z+i)$$

$$\int_{C^+_2(0)} \frac{A}{z-1}+\frac{B}{z+1}+\frac{C}{z-i}+\frac{D}{z+i}\,dz$$

$$=2\pi i\,(A+B+C+D)$$

$$\int_{C^+_{1/2}(-i)} \frac{1}{z^4-1}\,dz = 2\pi i \times D$$