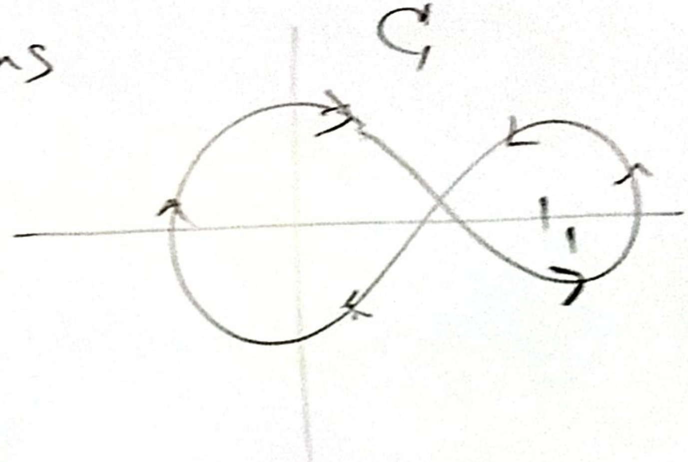
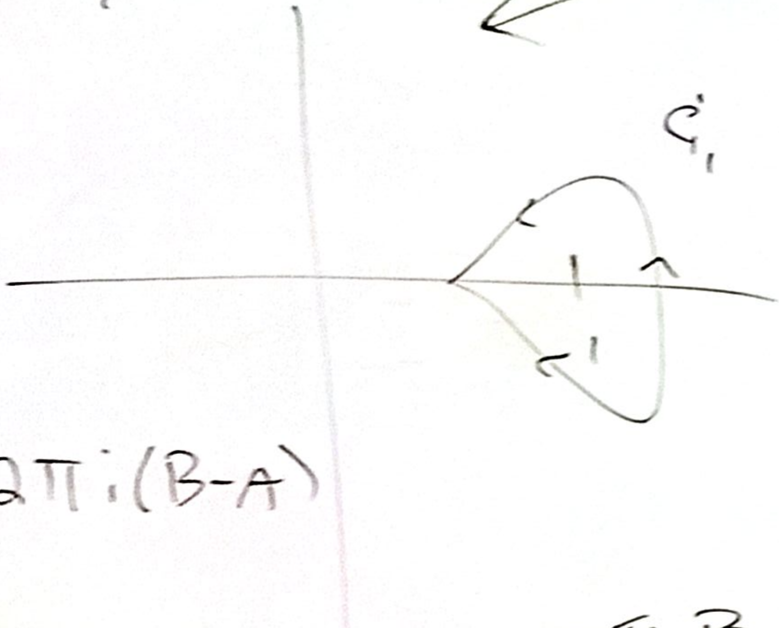
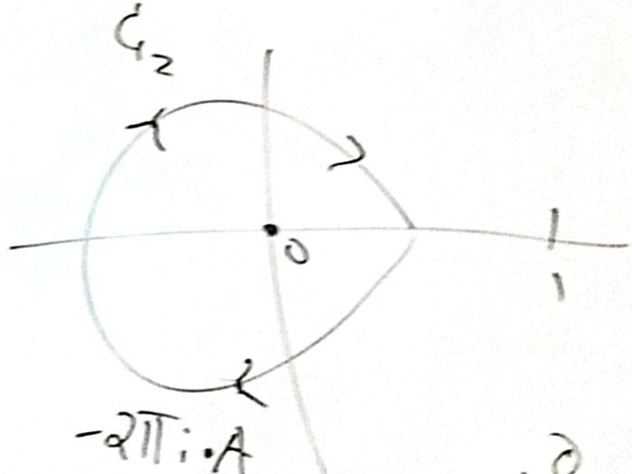
$$\int_{C} \frac{1}{z^2-z}\,dz \longrightarrow \text{partial fractions}$$







$$= 2\pi i(B-A)$$

$$C = C_1 + C_2$$

$$\int_{C_1} \cancel{\frac{A}{z}}^{\,\to 0} + \overbrace{\frac{B}{z-1}}^{2\pi i\cdot B}\,dz + \int_{C_2} \overbrace{\frac{A}{z}}^{-2\pi i\cdot A} + \cancel{\frac{B}{z-1}}^{\,\to 0}\,dz$$

$$\int_a^b f(x)\,dx \longrightarrow \int_{C} f(z)\,dz$$

$$= \int_a^b f(z(t))\,\frac{dz}{dt}\,dt$$

From real analysis

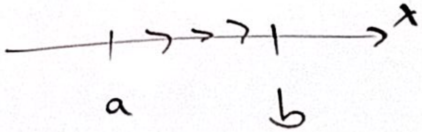
Define $F(x) = \int_{x_0}^{x} f(s)\,ds$

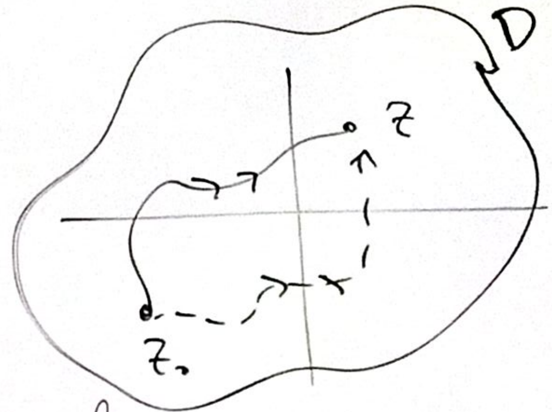
then $F'(x) = f(x)$

$$\int_a^b f(x)\,dx = F(b) - F(a), \quad \text{where } F'(x) = f(x)$$

D is simply connected

$$F(z) = \int_{z_0}^{z} f(s)\,ds$$





if the path taken from z_0 to z is contained inside a domain D on which $f(z)$ is analytic.

$$\int_{z_0}^{z_1} f(z)\,dz = F(z_1) - F(z_0)$$

provided $z_0, z_1 \in D$, and the path between is contained in D.

$$\int_{C_1} \cos(z)\,dz \quad , \quad C_1: \text{line segment from } -i \text{ to } 1+i$$

$$\int_0^1 \cos(z(t))\,z'(t)\,dt \qquad z(t) = -i(1-t) + t\cdot(1+i)$$

$$z'(t) = 1$$

$u = z(t)$

$du = z'(t)\,dt$

$$\int_{-i}^{1+i} \cos(u)\,du = \sin(u)\Big|_{-i}^{1+i}$$

$$= \sin(1+i) - \sin(-i)$$

Example 6.20

$$\int_C \frac{1}{z}\,dz$$

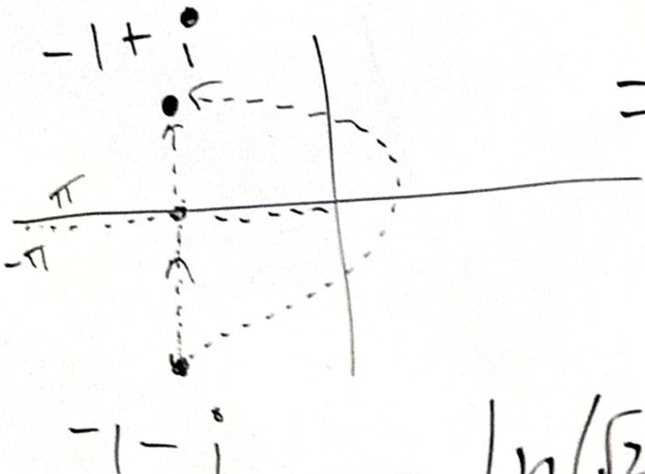
C: a path from $z_0 = -1-i$ to $z_1 = -1+i$

$$\frac{d}{dz}\operatorname{Log}(z) = \frac{1}{z}$$

$$= \operatorname{Log}(z_1) - \operatorname{Log}(z_0)$$

$$= \cancel{\ln(|-1+i|)} + i\operatorname{Arg}(-1+i)$$

$$- \cancel{\ln(|-1-i|)} - i\operatorname{Arg}(-1-i)$$



$$= \ln(\sqrt{2}) - \ln(\sqrt{2}) + i(3\pi/4 + 3\pi/4)$$

$$= i \cdot \frac{3\pi}{2}$$

$$\int \frac{1}{x^2-x}\,dx = \int -\frac{1}{x} + \frac{1}{x-1}\,dx$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$1 = A(x-1) + B \cdot x$$

$x=1$: $1 = B$

$x=0$: $1 = -A$