6.5 #3 $\displaystyle\int_{C_1^+(1)} \frac{1}{z+1} \cdot \frac{1}{(z-1)^2}\, dz$

$$\frac{2\pi i}{n!} f^{(n)}(z_0) = \int_C \frac{f(\xi)}{(\xi - z_0)^{n+1}}\, d\xi$$

Let:

$f(\xi) = \dfrac{1}{\xi+1}$

Then

$$\int_{C_1^+(1)} \frac{1}{z+1} \frac{1}{(z-1)^2} dz = \int_{C_1^+(1)} \frac{f(\xi)}{(\xi-1)^{1+1}} d\xi$$

$f'(\xi) = \dfrac{-1}{(\xi+1)^2}$

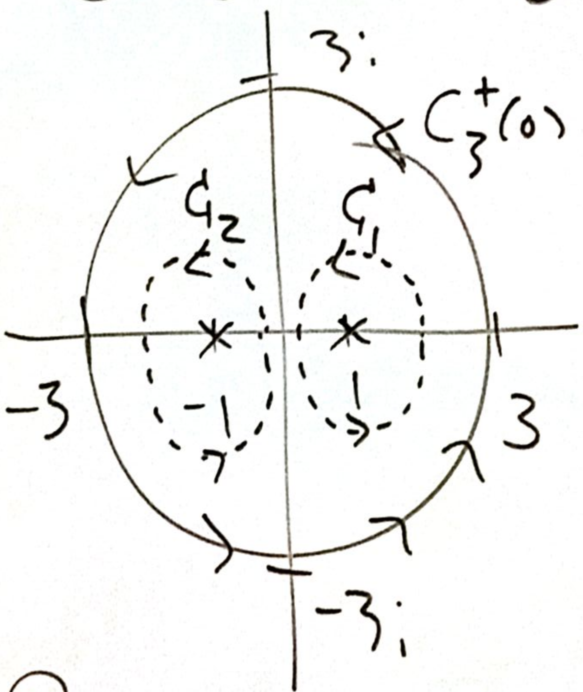
$$= \frac{2\pi i}{1!} f'(1) = -\frac{\pi}{2} i$$

6.5 #3 $\displaystyle\int_{C_1^+(-1)} \frac{1}{z+1} \cdot \frac{1}{(z-1)^2}\, dz$

$$f(\xi) = \frac{1}{(\xi-1)^2}$$

$$= \int_{C_1^+(-1)} \frac{f(\xi)}{\xi-(-1)}\, d\xi = \frac{2\pi i}{0!} f(-1) = \frac{\pi}{2} i$$

6.5 #3 $\displaystyle\int_{C_3^+(0)} \frac{1}{z+1}\cdot\frac{1}{(z-1)^2}\,dz$



$$\int_{C_3^+(0)} \frac{f(\xi)}{(\xi-z_0)^{n+1}}\,d\xi$$

$$\int_{C_3^+(0)} = \int_{C_1} + \int_{C_2} = -\frac{\pi}{2}i + \frac{\pi}{2}i$$

$$= 0$$

6.5 #9 $\displaystyle\int_{C_1^+(0)} \frac{e^z}{z^n}\,dz \qquad n \in \mathbb{N}$

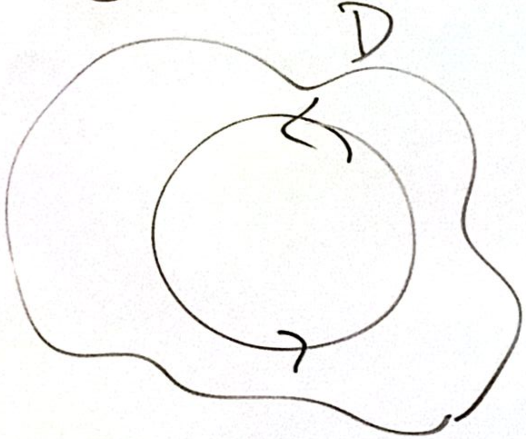
$f(\xi) = e^{\xi}$

$z_0 = 0$

$$\int_{C_1^+(0)} \frac{e^z}{z^n}\,dz = \int_{C_1^+(0)} \frac{f(\xi)}{(\xi-0)^{(n-1)+1}}\,d\xi = \frac{2\pi i}{(n-1)!} f^{(n-1)}(0)$$

$$= \frac{2\pi i}{(n-1)!}$$

6.5 $\displaystyle\int_{C_1^+(0)} z^n e^z \, dz \qquad n \in \mathbb{N}$

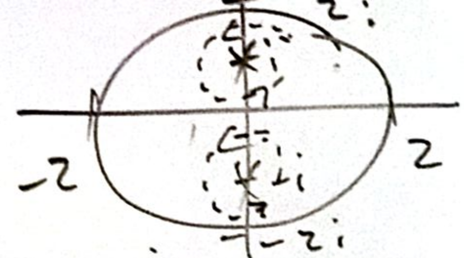


$= 0$

Since $f(z)$ is analytic on domain D which is simply connected and

$C_1^+(0) \subseteq D$

6.5 #13 $\displaystyle\int \frac{\sin(z)}{z^2+1}\,dz$



$z^2+1=(z-i)(z+i)$

$C_1^+(i)$

$f(\xi)=\dfrac{\sin(\xi)}{\xi+i}$

$\sin(z)=\dfrac{e^{iz}-e^{-iz}}{2i}$

$\displaystyle\int_{C_1^+(i)} \frac{f(\xi)}{(\xi-i)^{0+1}}\,d\xi=\frac{2\pi i}{0!}\,f(i)$

$=\dfrac{e^{-1}-e^{1}}{2i}$

$=2\pi i\cdot\sin(i)$

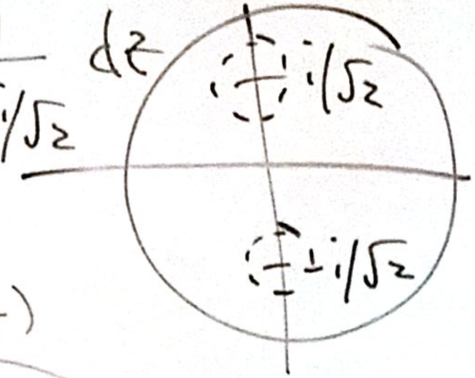
$=i\sinh(1)$

Thm 6.12

$=-2\pi\sinh(1)$

6.3

#1 (a) $\displaystyle\int_{C_1^+(0)} \frac{z}{2z^2+1}\,dz = \int_{C_1^+(0)} \frac{z}{2}\,\frac{1}{z+i/\sqrt{2}}\cdot\frac{1}{z-i/\sqrt{2}}\,dz$



$$= \int_{C_{1/2}(i/\sqrt{2})} \overbrace{\frac{z}{2(z+i/\sqrt{2})}}^{f(z)}\cdot\frac{1}{z-i/\sqrt{2}}\,dz + \int_{C^+_{1/2}(-i/\sqrt{2})} \overbrace{\frac{z}{2(z-i/\sqrt{2})}}^{f(z)}\cdot\frac{1}{z-(-i/\sqrt{2})}\,dz$$

$$= \frac{2\pi i}{0!}\,\frac{\cancel{i/\sqrt{2}}}{2\cdot 2\cancel{i/\sqrt{2}}} + \frac{2\pi i}{0!}\,\frac{\cancel{-i/\sqrt{2}}}{2\cdot(\cancel{-}2\cancel{i/\sqrt{2}})}$$

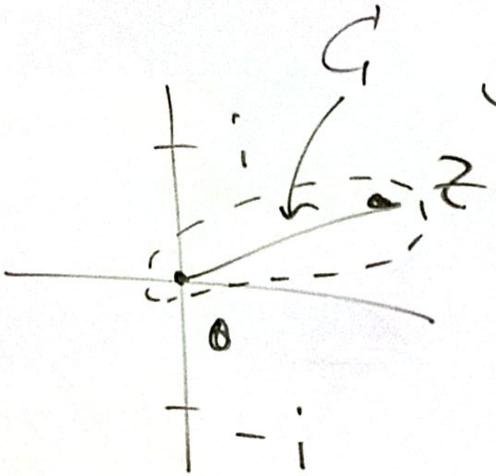
$$= \frac{2\pi i}{4} + \frac{2\pi i}{4} = \pi i$$

6.4 #18

$$\int_0^x \frac{1}{\xi^2+1}\,d\xi = \tan^{-1}(\xi)\Big|_0^x$$

$$= \tan^{-1}(x) - \tan^{-1}(0)$$

$$= \tan^{-1}(x)$$



Re(z)>0

$$\int_{C_1} \frac{1}{(\xi-i)}\,\frac{1}{(\xi+i)}\,d\xi$$

$$= \int_{C_1} \frac{A}{\xi-i}\,d\xi + \int_{C_1} \frac{B}{\xi+i}\,d\xi$$

$$\to A\cdot \operatorname{Log}(\xi-i)\Big|_0^z$$

$$= A\cdot\left(\operatorname{Log}(z-i) - \operatorname{Log}(-i)\right)$$

$$\underbrace{\cancel{\ln(|-i|)}}_{\to 0} + \underbrace{i\operatorname{Arg}(-i)}_{-i\frac{\pi}{2}}$$