1(a) $P(z) = z^4 + 4$

$$= (z - z_1)(z - z_2)(z - z_3)(z - z_4)$$

where $P(z_i) = 0 \quad \forall\ 1 \le i \le 4$

$$P(z) = (z^2 - 2i)(z^2 + 2i)$$

$$z^4 = -4 = 4e^{i(\pi + 2\pi k)}$$

$$z = 4^{1/4} e^{i\left(\frac{\pi}{4} + \frac{\pi k}{2}\right)} \qquad k \in \{0,1,2,3\}$$

$$P(z) = (z - \sqrt{2}e^{i\pi/4})(z - \sqrt{2}e^{i\frac{3\pi}{4}})(z - \sqrt{2}e^{i5\pi/4})$$

$$(z - \sqrt{2}e^{i7\pi/4})$$

1(b) $P(z) = z^2 + (1+i)z + 5i = (z-(1-2i))*(z-(-2+i))$

$$z = \frac{-(1+i) \pm \sqrt{(1+i)^2 - 4\cdot 5i}}{2}$$

$$= \frac{-(1+i) \pm \sqrt{-18i}}{2}$$

$$= \frac{-1-i \pm (3-3i)}{2}$$

$$= \frac{-1 \pm 3 + i(-1 \pm (-3))}{2}$$

$$= 1-2i, \; -2+i$$

$\sqrt{18 e^{-i\pi/2}}$

$3\sqrt{2}\, e^{-i\pi/4}$

$3\sqrt{2}(\cos(\pi/4) - i\sin(\pi/4))$

$3\sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$

$3-3i$

2. $f(z) = az^n + b$ $\qquad R = \{ z \mid |z| \le 1 \}$

Show $\max_{z \in R} |f(z)| = |a| + |b|$, $n > 0$, $a, b \in \mathbb{C}$

$f(z)$ is analytic on R, continuous on

boundary

∂R for $|z| = 1$

$$
\begin{aligned}
|f(z)| &= |az^n + b| \\
&\le |az^n| + |b| \\
&= |a|\,|z^n| + |b| \\
&= |a|\,|z|^n + |b| \\
&= |a| + |b|
\end{aligned}
$$

2. $|f(z)| \le |a| + |b|$

$f(z) = az^n + b$ for $|z| = 1$, $z = e^{i\theta}$

$= a e^{in\theta} + b$ $\quad |a| = \alpha$

$|b| = \beta$

$\theta_a + n\theta = \theta_b$ $\quad = \alpha e^{i\theta_a} e^{in\theta} + \beta e^{i\theta_b}$

$\theta = \frac{\theta_b - \theta_a}{n}$ $\quad = \alpha e^{i(\theta_a + n\theta)} + \beta e^{i\theta_b}$ $\quad \alpha, \beta > 0$

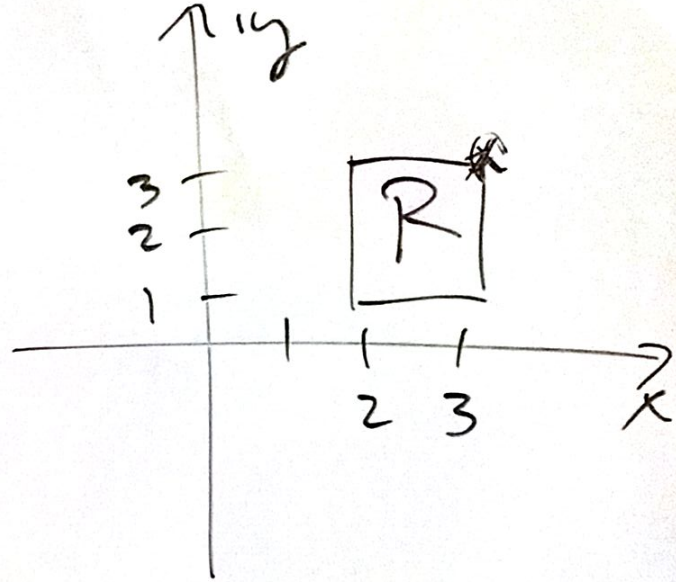
$= \alpha e^{i\theta_b} + \beta e^{i\theta_b}$

$= (\alpha + \beta) e^{i\theta_b}$

$|f(z)| = \alpha + \beta = |a| + |b|$

4 (a) $f(z) = z^2$

$$\max_{z \in R} |f(z)|$$



$$|f(z)| = |z^2| = |z|^2$$

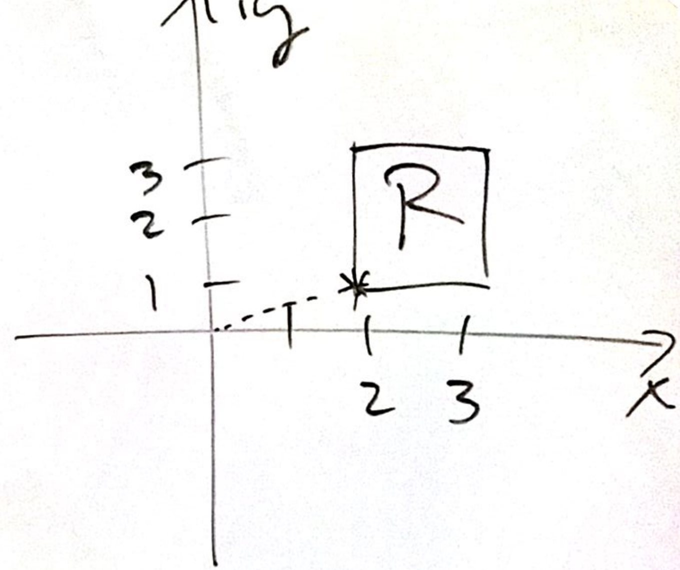
$$= \sqrt{x^2+y^2}^{\,2} = x^2+y^2$$

max occurs @ $z = 3+3i$

$$\max_{z \in R} |f(z)| = 18$$

4 (b) $f(z) = z^2$

$\min_{z \in R} |f(z)|$



minum of $|f(z)|$ occurs

where the max of $\frac{1}{|f(z)|}$ occurs