

Math 1303 - Math in the Liberal Arts

Exam 1 Solutions

1. Construct truth tables for the following statements:

(a) $[(p \rightarrow q) \wedge (p \rightarrow r)] \leftrightarrow [p \rightarrow (q \wedge r)]$

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$	$q \wedge r$	$p \rightarrow (q \wedge r)$	the whole thing
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

(b) $(p \vee q \vee r) \leftrightarrow [(\sim p \wedge \sim q) \rightarrow r]$

p	q	r	$p \vee q$	$p \vee q \vee r$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$(\sim p \wedge \sim q) \rightarrow r$	the whole thing
T	T	T	T	T	F	F	F	T	T
T	T	F	T	T	F	F	F	T	T
T	F	T	T	T	F	T	F	T	T
T	F	F	T	T	F	T	F	T	T
F	T	T	T	T	T	F	F	T	T
F	T	F	T	T	T	F	F	T	T
F	F	T	F	T	T	T	T	T	T
F	F	F	F	F	T	T	T	F	T

For problems 2–5, let p , q , r , and s be the following sentences:

p : There is no open water on the planet Arrakis.

q : Arrakis is a desert planet.

r : Sandworms live in the desert.

s : Arrakis is the only planet on which sandworms thrive.

2. Use p , q , r , and s as above to write each of the following symbolic statements in words.

(a) $q \rightarrow p$

If Arrakis is a desert planet, then there is no open water on Arrakis.

(b) $r \wedge s$

Sandworms live in the desert and Arrakis is the only planet on which sandworms thrive.

(c) $(r \wedge s) \rightarrow q$

If sandworms live in the desert and Arrakis is the only planet on which sandworms thrive, then Arrakis is a desert planet.

(d) $s \leftrightarrow (p \wedge q)$

Arrakis is the only planet on which sandworms thrive if, and only if, there is no open water on the planet Arrakis and Arrakis is a desert planet.

3. State the negation of the following sentence without using a conditional sentence form:

If spice melange can be found in the desert, then sandworms also live in the desert.

This sentence is of the form $p \rightarrow q$, to compute its negation, we need to use the fact that $(p \rightarrow q) \Leftrightarrow (\sim p \vee q)$. We can now negate:

$$\begin{aligned}\sim (p \rightarrow q) &\Leftrightarrow \sim (\sim p \vee q) \\ &\Leftrightarrow p \wedge \sim q\end{aligned}$$

Therefore, the negation of our sentence becomes:

The spice melange can be found in the desert and sandworms do not live in the desert.

4. State the negation of the following sentence without using a conjunction:

Walk across the desert in an irregular pattern and conserve your water carefully.

This sentence is of the form $p \wedge q$, and we know $\sim (p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$, i.e. the negation of the conjunction is the disjunction of the negation:

Do not walk across the desert in an irregular pattern or do not conserve your water carefully.

5. Consider the following argument:

If Arrakis is a desert planet, then Arrakis is the only planet on which sandworms thrive. If there is no open water on Arrakis, then sandworms live in the desert. Arrakis is a desert planet or sandworms do not live in the desert. Therefore, Arrakis is the only planet on which sandworms thrive or there is some open water on Arrakis.

(a) Translate the above argument into symbolic form using the definition of p , q , r , and s as defined previously.

Using the definitions of p , q , r , and s as given we have:

$$\begin{array}{l}q \rightarrow s \\ p \rightarrow r \\ \underline{q \vee \sim r} \\ \therefore s \vee \sim p\end{array}$$

(b) Determine if the argument is valid or a fallacy.

As a logical sentence, the above argument is $[(q \rightarrow s) \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow (s \vee \sim p)$. On the top of the next page, the final column in the truth table consists only of all 'True's. The sentence is a tautology and thus the argument is valid.

p	q	r	s	Δ $q \rightarrow s$	\square $p \rightarrow r$	$\sim r$	\diamond $q \vee \sim r$	$\Delta \wedge \square$	\star $\Delta \wedge \square \wedge \diamond$	$\sim p$	\blacklozenge $s \vee \sim p$	$\star \rightarrow \blacklozenge$
T	T	T	T	T	T	F	T	T	T	F	T	T
T	T	T	F	F	T	F	T	F	F	F	F	T
T	T	F	T	T	F	T	T	F	F	F	T	T
T	T	F	F	F	F	T	T	F	F	F	F	T
T	F	T	T	T	T	F	F	T	F	F	T	T
T	F	T	F	T	T	F	F	T	F	F	F	T
T	F	F	T	T	F	T	T	F	F	F	T	T
T	F	F	F	T	F	T	T	F	F	F	F	T
F	T	T	T	T	T	F	T	T	T	T	T	T
F	T	T	F	F	T	F	T	F	F	T	T	T
F	T	F	T	T	T	T	T	T	T	T	T	T
F	T	F	F	F	T	T	T	F	F	T	T	T
F	F	T	T	T	T	F	F	T	F	T	T	T
F	F	T	F	T	T	F	F	T	F	T	T	T
F	F	F	T	T	T	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T	T	T	T

6. Use a Venn diagram to determine if the following syllogism is valid or is a fallacy.

Some people live in the desert of Arrakis.
Some people who live on Arrakis are Fremen.
All Fremen live in the desert.
Some people who live on Arrakis can ride sandworms.
All Fremen can ride sandworms.
 \therefore *Only those who live in the desert can ride sandworms.*

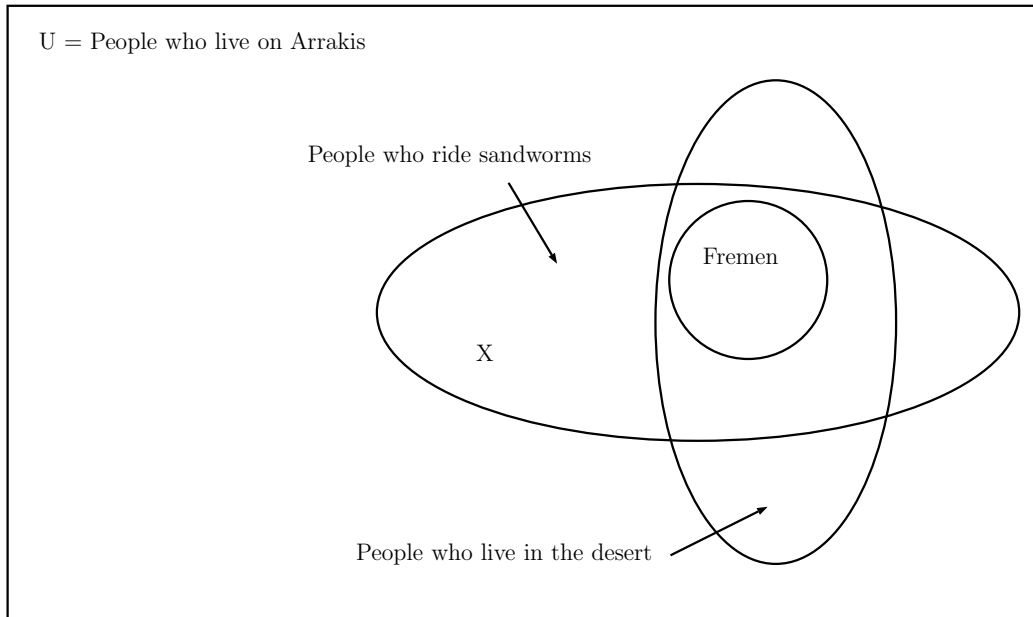


Figure 1: Venn diagram for the syllogism given in problem 6.

The 'X' in the above diagram shows that syllogism is invalid, as there are people who do not live in the desert who can ride sandworms (see the x above).