

Math 1303 - Math in the Liberal Arts

Exam 2 Solutions

1. Consider the following set, R , expressed in set-builder notation:

$$R = \{x \mid x \text{ is a current United States Representative from Oklahoma.}\}$$

- (a) Express the set R in roster notation:

$$\{\text{Kevin Hern, Markwayne Mullin, Frank Lucas, Tom Cole, Kendra Horn}\}$$

- (b) What is $n(R)$?

$$n(R) = 5$$

2. C is defined in roster as follows:

$$C = \{\text{Oden Grube, Steve Brittingham, Jerry Tomlinson, Mike Morris, Danny Sherrer}\}$$

- (a) Express the set C in set builder notation.

$$C = \{x \mid x \text{ is a current member of the city council of Durant, Oklahoma.}\}$$

- (b) Compute $n(C)$.

$$n(C) = 5$$

3. Describe in a full English sentence the follow set: $P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

P is the set of all prime numbers less than 20.

4. Let $A = \{1, 2, 3, \{1, 2\}, \{1\}\}$. Determine if each of the following statements are true or false.

(a) $1 \in A$ - true

(b) $\{1\} \in A$ - true

(c) $\{1, 2\} \in A$ - true

(d) $\{1, 2\} \subseteq A$ - true

(e) $\{\{1, 2\}\} \subseteq A$ - true

(f) $3 \in A$ - true

(g) $3 \subseteq A$ - false

(h) $\{\} \in A$ - false

(i) $\{\} \subseteq A$ - true

5. Compute all possible subsets of $L = \{\square, \diamond, \triangle\}$

Since $n(L) = 3$, there will be $2^3 = 8$ subsets of L :

$$\{\}, \{\square\}, \{\diamond\}, \{\triangle\}, \{\square, \diamond\}, \{\square, \triangle\}, \{\diamond, \triangle\}, \{\square, \diamond, \triangle\}$$

6. For this problem, the sets O , C , and L will be defined as follows:

$$O = \{x \mid x \text{ is a city in Oklahoma.}\}$$

$$C = \{x \mid x \text{ is a capital of a state in the USA.}\}$$

$$L = \{x \mid x \text{ is a city in with at least 500,000 people.}\}$$

Answer the following questions. Please cite where you obtained your information from to answer them.

For these answers, the following links were used:

[https://en.wikipedia.org/wiki/List_of_United_States_cities_by_population.](https://en.wikipedia.org/wiki/List_of_United_States_cities_by_population)

https://en.wikipedia.org/wiki/List_of_cities_and_towns_in_Oklahoma

https://en.wikipedia.org/wiki/List_of_capitals_in_the_United_States

(a) Compute $O \cap C$.

$O \cap C$ is the city in Oklahoma which is also that state capital of Oklahoma:

$$O \cap C = \{ \text{Oklahoma City} \}$$

(b) Compute $C \cap L$.

$O \cap C =$

{Phoenix, Austin, Columbus, Indianapolis, Denver, Boston, Nashville, Sacramento, Atlanta, Oklahoma City}

Note that the cities of Sacramento and Atlanta had populations less than 500,000 at the time of the last census, but the estimates for the populations in 2019 was about 500,000.

(c) Compute $O \cap L$.

$O \cap L$ is the cities in Oklahoma with a population of at least 500,000 people:

$$O \cap L = \{ \text{Oklahoma City} \}$$

(d) Relate $O \cap C$ to $C \cap L$ using \subseteq or $=$. Also relate $O \cap C$ to $O \cap L$ using \subseteq or $=$.

$O \cap C \subseteq C \cap L$ and $O \cap C = O \cap L$

(e) What would the population in the definition of L need to be so that $C \subset L$?

The capital of Vermont, Montpelier, has an estimated population of 7,855 as of 2019, and is the state capital with the smallest population. Therefore, we would have to set the value to anything less than or equal to 7,855.

7. Let $A = \{0, 1, 2, \dots\}$, and $B = \{10, 20, 30, \dots\}$. Is $n(A) < n(B)$, $n(A) = n(B)$, or $n(A) > n(B)$?

Both sets have the exact same cardinal number, which is infinity (the same degree of infinity in this case).

8. Let the sets A and B be defined as

$$A = \{x \mid x \text{ is a student in Math 1303 who studied for the exam using their notes.}\}$$

$$B = \{x \mid x \text{ is a student in Math 1303 who studied for the exam using the textbook.}\}$$

There are 33 students enrolled in our course. Of those, 14 students stated that they used the textbook to study for the exam, while 17 used their notes to study. Of those polled, 7 said that they did not study at all for the exam.

(a) Construct a Venn diagram for the sets A and B .

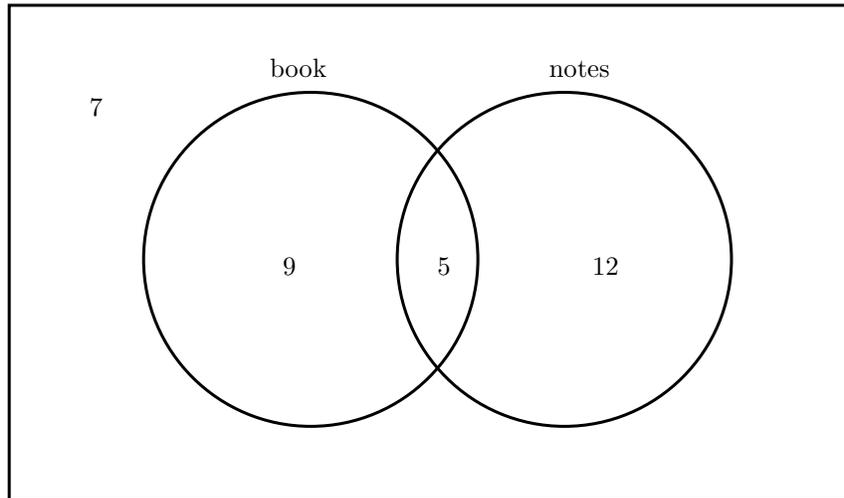


Figure 1: Venn diagram for the syllogism given in problem 6.

(b) How many students used both their notes and the textbook to study for the exam?

There were 5 students in $A \cap B$, which is the region in question.

(c) How many students did not use the textbook to study for the exam?

This would be the cardinality of B' , which is 19.

(d) How many students did not use both their notes and the textbook to study?

This would be the cardinal number of the set $(A \cap B)'$, which is 28.

9. Students were interviewed and asked about three facets of their student experience: if they lived on campus, were a student athlete, and belonged to the Honors program. Here were the results:

- 73 were student athletes only.
- 45 were Honors students.
- 32 were Honors students only.
- 533 lived on campus.
- 102 student athletes lived on campus.
- 4 lived on campus, were student athletes, and were Honors students.
- 12 were Honors students living on campus.
- 261 interviewed were none of the three.

(a) Construct a three set Venn diagram for the given information.

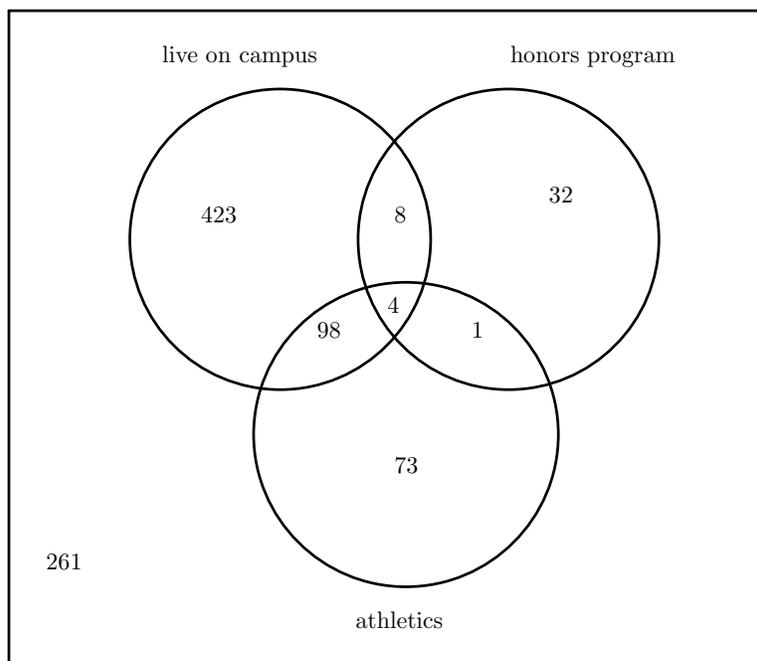


Figure 2: Venn diagram for the syllogism given in problem 6.

(b) How many total students were interviewed?

There were 900 total students interviewed if we add up all 8 regions of the Venn diagram.

(c) How many students were in the Honors program, and were also student athletes, but did not live on campus?

Only 1 student satisfied that criteria.

(d) How many students did not live on campus?

There were 367 students in the complement to the region 'live on campus'.

(e) How many students belonged to exactly two of the given groups?

There are three regions which overlap exactly two sets, with totals 98, 1, and 8, totaling 107.