Math 1303 - Math in the Liberal Arts
Exam 3 Solutions

For problems 1–8, we are going to let $a = 450,450$, $b = 97,020$, and $c = 67,914,000$. The prime factorizations of a, b, and c are as follows:

$$a = 2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13, \quad b = 2^2 \cdot 3^2 \cdot 5 \cdot 7^2 \cdot 11, \quad c = 2^3 \cdot 3^2 \cdot 5^2 \cdot 7^3 \cdot 11$$

Use this information to answer problems 1–8.

1. Compute the GCD(a, b), leave your answer in the prime factorization form (DO NOT MULTIPLY OUT!).

$$\text{GCD}(a, b) = 2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$$

2. Compute the GCD(a, c), leave your answer in the prime factorization form (DO NOT MULTIPLY OUT!).

$$\text{GCD}(a, c) = 2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11$$

3. Compute the GCD(a, b, c), leave your answer in the prime factorization form (DO NOT MULTIPLY OUT!).

$$\text{GCD}(a, c) = 2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$$

4. Compute the LCM(a, b), leave your answer in the prime factorization form (DO NOT MULTIPLY OUT!).

$$\text{LCM}(a, b) = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13$$

5. Compute the LCM(b, c), leave your answer in the prime factorization form (DO NOT MULTIPLY OUT!).

$$\text{LCM}(b, c) = 2^3 \cdot 3^2 \cdot 5^2 \cdot 7^3 \cdot 11$$

6. Compute the LCM(a, b, c), leave your answer in the prime factorization form (DO NOT MULTIPLY OUT!).

$$\text{LCM}(b, c) = 2^3 \cdot 3^2 \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13$$

7. Write a/b as a rational number in reduced form, where the numerator and denominator are still expressed in fully factored form.

$$\begin{aligned}
\frac{a}{b} &= \frac{450,450}{97,020} \\
&= \frac{2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13}{2^2 \cdot 3^2 \cdot 5 \cdot 7^2 \cdot 11} \\
&= \frac{3^{2-2} \cdot 5^{2-1} \cdot 11^{1-1} \cdot 13}{2^{2-1} \cdot 7^{2-1}} \\
&= \frac{5 \cdot 13}{2 \cdot 7}
\end{aligned}$$

8. Express $a \cdot (b \cdot c)$ in prime factorization form.

$$\begin{aligned}
a \cdot (b \cdot c) &= (2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13) \cdot ((2^2 \cdot 3^2 \cdot 5 \cdot 7^2 \cdot 11) \cdot (2^3 \cdot 3^2 \cdot 5^2 \cdot 7^3 \cdot 11)) \\
&= (2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13) \cdot ((2^2 \cdot 2^3 \cdot 3^2 \cdot 3^2 \cdot 5 \cdot 5^2 \cdot 7^2 \cdot 7^3 \cdot 11 \cdot 11)) \\
&= (2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13) \cdot (2^5 \cdot 3^4 \cdot 5^3 \cdot 7^5 \cdot 11^2) \\
&= 2 \cdot 2^5 \cdot 3^2 \cdot 3^4 \cdot 5^2 \cdot 5^3 \cdot 7 \cdot 7^5 \cdot 11 \cdot 11^2 \cdot 13 \\
&= 2^6 \cdot 3^6 \cdot 5^5 \cdot 7^6 \cdot 11^3 \cdot 13
\end{aligned}$$

9. Express $x = 101.215$ as a rational number. You do not have to express it in reduced form.

$$101.215 = \frac{101,215}{1000}$$

10. Express $x = 0.\overline{215}$ as a rational number. You do not have to express it in reduced form.

Notice that if $x = 0.\overline{215}$, then $1000x = 215.\overline{215}$, thus

$$\begin{aligned} 1000x - x &= 215.\overline{215} - 0.\overline{215} \\ 999x &= 215 \\ x &= \frac{215}{999} \end{aligned}$$

11. Consider the following sequence of numbers: $\{-5, -1, 3, 7, 11, 15, \cdots, 327, 331\}$.

(a) Is the sequence arithmetic, geometric, or neither?

The difference between successive numbers is a constant value $d = 4$, so it is arithmetic.

(b) How many numbers are there in the sequence listed above?

Since we go up by a constant value of 4, if we start at -5, we must go 84 steps of length 4 to get to 331, thus there are 85 numbers in the sequence.

(c) Compute the sum of the numbers in sequence, you do not have to simplify your answer.

Using the answers from parts (a) and (b), we have

$$S_{85} = \frac{85 \cdot (331 - 5)}{2} = \frac{85 \cdot 326}{2} = 13855$$

12. Consider the following geometric sequence of numbers: $\left\{2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \frac{32}{81}, -\frac{64}{243}, \frac{128}{729}\right\}$.

(a) Find the value of r which makes the sequence geometric.

Remember that r is the common ratio between successive terms in the sequence, so $r = (-\frac{4}{3})/2 = -\frac{2}{3}$.

(b) Find the sum of the 7 terms in the geometric sequence given. You do not have to simplify your answer.

$$S_7 = \frac{2 \cdot \left(1 - (-2/3)^7\right)}{1 - (-2/3)} = \frac{926}{729}$$